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ABSTRACT

This booklet contains a representative sample of the efforts of colleagues at 11 institutions to use graphing calculators to enhance the teaching of calculus and precalculus. The first section contains examples of graphs for teachers to choose from for presentations, including: simple examples to illustrate some standard ideas in precalculus, examples of graphs for which the window choice is critical and a knowledge of mathematics is essential for predicting hidden behavior, and examples that produce interesting shapes. The next sections contain generic and machine specific worksheets for calculus and precalculus. Next is a section that contains original programs for both the TI-81 and HP-48S calculators. A section to acquaint students with the use of the TI-81, HP-48G, and HP-48S concludes the booklet. (MKR)

SELECTED MATERIALS

from a

CALCULATOR ENHANCED INSTRUCTION PROJECT

by an

EXPANDED CONSORTIUM

of NEW JERSEY and PENNSYLVANIA

EDUCATIONAL INSTITUTIONS

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Fall 1994
DUE-9252491

This booklet contains a representative sample of the efforts of colleagues at eleven institutions to use graphing calculators to enhance the teaching of calculus and precalculus. We have included worksheets suitable for duplication . Some are machine specific (TI-81, HP-48S) while others are generic. There is a "how to" section to acquaint students with the use of the TI-81 and HP-48S. There is a section of "favorite functions" which provides stimulating classroom examples and a section of handy programs.

We would particularly like to thank the **NATIONAL SCIENCE FOUNDATION ,Division of Undergraduate Education** for funding this project and James Lightbourne, Program Officer, for his direction and encouragement.

We would like to thank the following consortium members for major contributions to this booklet: Barbara Brook, Gerald Colabelli, George Evanovich, Mark Galit, Bruce Hoelter, Martin Johnson, Charles Miller, William Scott, Fay Sewell, Michael Sheehy, Nora Thornber, Guy Vuotto, Roger Willig, and Raymond Zenere. Our gratitude is also extended to Lynda Mee, Richard Lucas, Wayne Ackerman, and the administrations of Union C.C. and of the other ten institutions for their support and cooperation.

Sincerely,

Jean Lane

Jean Lane
Director

Elaine Petsu

Elaine Petsu
Co-director

TABLE OF CONTENTS

Originally funded by the National Science Foundation for the academic year 1991-92 (USE 9153258), ten participants from five New Jersey community colleges attended workshops taught by Dr. John Kenelly of Clemson University and Dr. Thomas Tucker of Colgate University on the use of the TI-81 and HP-48 graphics calculators to enhance instruction of precalculus and calculus topics. The participants then worked individually, in pairs at their own institutions, and as a group to plan implementation.

During the spring semester 1992, one precalculus and one calculus course were taught at each institution using calculators provided by the grant. Students surveyed at semester's end were generally enthusiastic, as were their instructors. Advantages cited included the ability to solve harder, more realistic problems; the ability to concentrate on concepts rather than monotonous computations that could be done by the machine; and the ease and clarity of visualization afforded by the graphs.

The Consortium was expanded to eleven institutions including two-year and four-year colleges, and high schools. The National Science Foundation, Division of Undergraduate Education provided funding for the project for the 1992-93 and 1993-94 academic years (DUE 9252491). The continuing participants introduced calculators into statistics, trigonometry, and second and third year calculus courses. Dr. Donald LaTorre of Clemson University conducted a workshop on Linear Algebra for the continuing participants and interested colleagues in April, 1993.

Activities for the new participants were similar to those from the original grant. Sixteen instructors chosen to participate for 1992-94, as well as invited faculty from the continuing institutions, attended an intensive, hands-on workshop October 2-4, 1992, using the TI-81 with Prof. Virginia Crisonino of Union County College as instructor. A second workshop on the HP-48 was presented October 14-16 by Project Director Jean Lane. A video tape of the HP workshop was made by Union County College's Media Center, and is available for use by the members of the Consortium.

The new members began implementation of calculator usage in their classes during Spring 1993. At the colleges, one precalculus and one calculus course were taught using graphics calculators. The high school participants began to introduce the technology into their classes as appropriate; their actual implementation began during the 1993-94 school year, again in precalculus and calculus.

In February, 1993, the new participants met with Dr. John Kenelly for additional guidance and inspiration. They were also invited to the Linear Algebra workshop with LaTorre. Toward the end of the semester, they did presentations about their experiences at their home institutions.

During the 1993-94 academic year, as implementation continued, all participants convened for two workshop/sharing sessions, one each semester. The group's enthusiasm ensures that such sharing sessions will continue into the 1994-95 academic year as well.

In order to disseminate the project's activities, Project Director Jean Lane and Co-Project Director Elaine Petsu editted instructional materials developed by the group for distribution to interested educators. Conferences and/or in-service workshops will take place at both Union County College and Raritan Valley Community College to ensure dissemination of project results to area high achools.

Participants in the project have made over thirty presentations, including workshops and poster sessions, to various groups as of October, 1994.

SEEING IS BELIEVING ... OR IS IT ?

Teachers using graphing calculators often find it useful to have a catalog of examples from which to quickly choose appropriate illustrations for presentations. The following graphs include

1. simple examples to illustrate some standard ideas in precalculus
2. examples of graphs for which the window choice is critical and a knowledge of mathematics is essential for predicting hidden behavior
3. examples that produce interesting shapes.

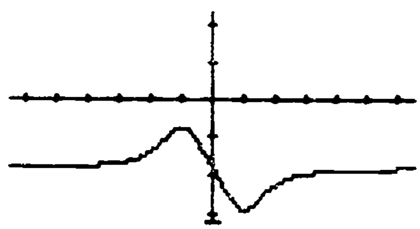
Windows given are for the TI-81 . The same windows used with a TI-82, TI-85, HP48S or HP48G can produce the same basic result in some instances but can produce very different looking results in others. Doing the examples which follow on several calculators for comparison can make one acutely aware of the need to use mathematical facts in conjunction with the graphs on the screen . Different screen sizes and differing numbers of pixels in rows and columns greatly affect results .

EXAMPLE 1

Can a rational function cross its horizontal asymptote ?

$$f(x) = \frac{-3x^4 - 3x - 3}{x^4 + 1}$$

This function approaches its horizontal asymptote $y = -3$ with two different end behaviors and its intersection with its horizontal asymptote can be obtained with minimal computation.



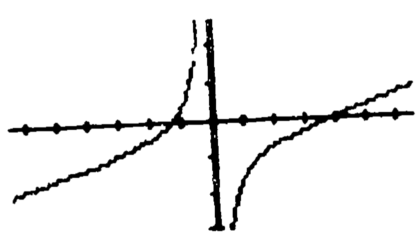
$-5 \leq x \leq 5$
$-5 \leq y \leq 5$

EXAMPLE 2

Are the asymptotes for a rational function always vertical or horizontal lines ?

$$f(x) = \frac{x^2 - 2x - 3}{2x}$$

This function has $y = \frac{1}{2}x - 1$ as an oblique asymptote and integral x-intercepts.

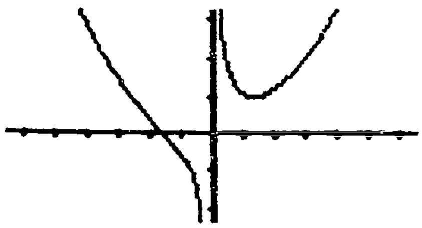


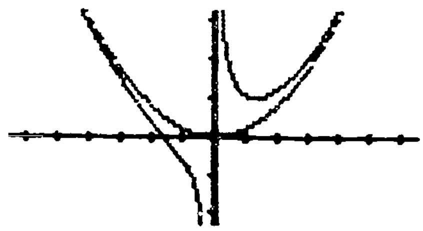
$-5 \leq x \leq 5$
$-5 \leq y \leq 5$

EXAMPLE 3
Are rational functions ever asymptotic to anything but lines?

$$f(x) = \frac{x^3 + 2}{x}$$

This function is asymptotic to $y = x^2$. The example can prompt a more general discussion of the asymptotic behavior of rational functions.



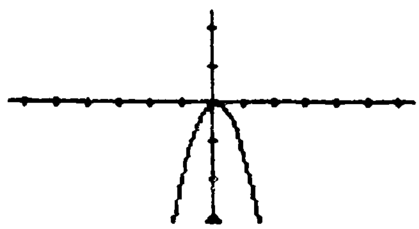


$-5 \leq x \leq 5$
$-10 \leq y \leq 10$

EXAMPLE 4
Why don't I see anything on my screen?

$$f(x) = (1/15)x^3 - 27x^2 + 15x - 15$$

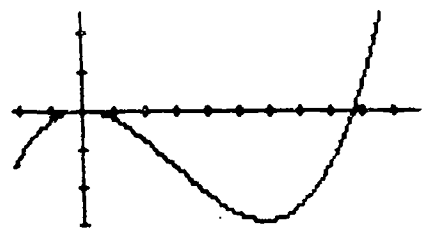
When viewed on a standard graphing window, this curve does a disappearing act. Using a trace key can offer some clues as to the choice of an appropriate window. Suggest that students try $-20 \leq x \leq 20$, $-500 \leq y \leq 500$ and then discuss the completeness of the picture.



$-20 \leq x \leq 20$
$-500 \leq y \leq 500$

The y-values for this cubic function where the curve's characteristic behavior occurs are much smaller than most students imagine when viewing the coefficients.

$-100 \leq x \leq 500$
$-700000 \leq y \leq 700000$



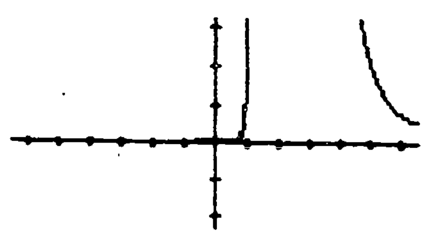
EXAMPLE 5

Why should I bother to memorize those asymptote rules when I've got a graphing calculator to do the work ?

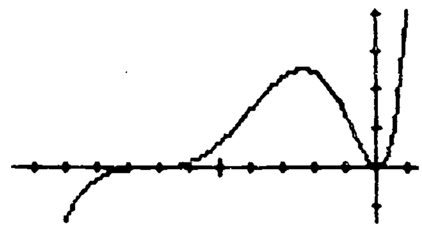
$$f(x) = \frac{x^2(x+1)^3}{(x-2)^2(x-4)^4}$$

Viewed on a standard graphing window, the function would seem to have a single vertical asymptote on the interval [2,4] yet theory predicts vertical asymptotes at $x = 2$ and $x = 4$.

$-10 \leq x \leq 10$
$-10 \leq y \leq 10$



Again , the problem is with the choice of an appropriate window. Only a knowledge of mathematical facts leads one to recognize the incompleteness of the graph. As in Example 4 , the y range required is much larger than most students anticipate ; however, even with an appropriate range, a totally satisfying single picture may be difficult to obtain . It is suggested that students consider separately the curve's behavior near 2 and then near 4 before combining all ideas into one graph for f(x). Windows like $1.8 \leq x \leq 2.2$, $0 \leq y \leq 70{,}000$ and $3.6 \leq x \leq 4.4$, $0 \leq y \leq 70{,}000$ may serve for beginning observations at each location . A graph showing these combined behaviors can be obtained using $1.4 \leq x \leq 4.4$, $0 \leq y \leq 40{,}000$ with refinements added from the localized observations. Students should be cautioned to investigate f(x) near its two roots -1 and 0 (where it is tangent to the axis) .



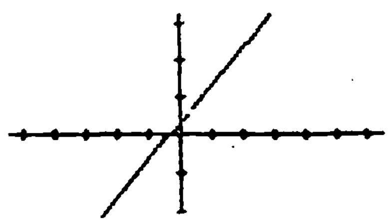
$1.4 \leq x \leq 4.4$
$0 \leq y \leq 40{,}000$

EXAMPLE 6
What hole?

$$f(x) = \frac{x^2 - 9}{x - 3}$$

When graphed on the standard window $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, f(x) appears to be a continuous function. Choosing a more "friendly" window , however, clearly exposes the hole in the graph. Using $0 \leq x \leq 9.5$, $0 \leq y \leq 6.4$ yields a good picture from a range whose choice is easy to explain in terms of pixels. (NOTE: for TI-82 use $0 \leq x \leq 9.4$, $0 \leq y \leq 6.5$
for HP48G use $-65 \leq x \leq 65$, $-32 \leq y \leq 32$)

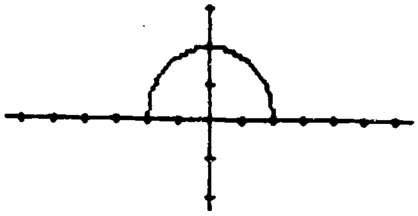
$0 \leq x \leq 9.5$
$0 \leq y \leq 6.4$



EXAMPLE 7
HOW did you do that ? I didn't expect that at all !!

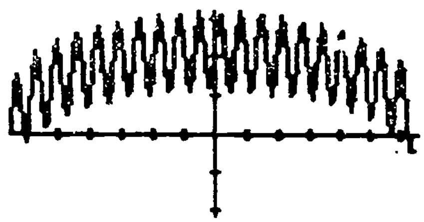
$$f(x) = \sin(10\pi x) + \sqrt{(4 - x^2)}$$

Viewed in a standard window $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, the graph of f(x) would seem to inspire few comments other than a mention of the range of f(x) as determined by the square root term.



$-10 \leq x \leq 10$
$-10 \leq y \leq 10$

Restrict the window to the function's domain or choose a standard trigonometric window and the curve seems to explode revealing the periodic influence of the sine term . Simple substitution determines the value of f(x) at the endpoints . Zooming out at the endpoints and comparing the results can pull in a discussion of symmetry (or lack of).

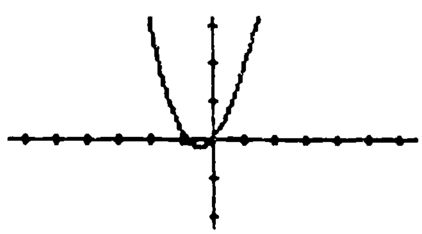


$-2 \leq x \leq 2$
$-3 \leq y \leq 3$

EXAMPLE 8
So why doesn't this graph "explode" ? (Calculus to the rescue)

$$f(x) = x^2 + \sin x$$

After an example like #7, students might suspect that any composite function containing a periodic function will exhibit some type of oscillating behavior . Using a standard window or trigonometric window, the graph of f(x) is not very exciting and successive zooms would indicate that the apparent behavior is the true behavior . But should that alone be a convincing argument ?



$-6.28 \leq x \leq 6.28$
$-3 \leq y \leq 3$

A more satisfying argument would rely on calculus. Since

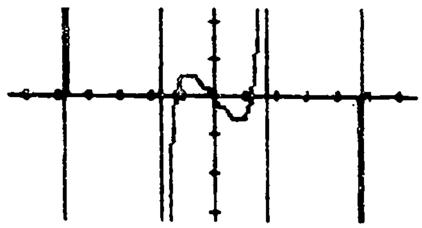
$f''(x) = 2 - \sin x$, $1 \leq f''(x) \leq 3$. Since this curve is always concaved upward , f(x) cannot exhibit the oscillations of the function in Example 7 .
A discussion of which term dominates in Example 7 and Example 8 could follow. Graphing $y = \sin(10\pi x)$ against $y = \sqrt{(4 - x^2)}$ and $y = x^2$ against $y = \sin x$ can be useful in this exploration.

EXAMPLE 9
Should I always use a trig friendly window when my function involves a trigonometric function ?

$$f(x) = \tan x - 2x$$

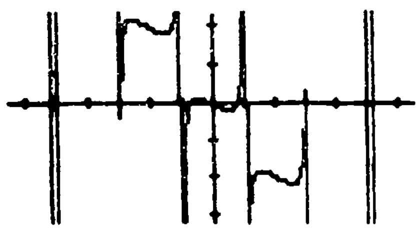
The context for this graph is finding the solutions of the equation tan x = 2x. When graphed on a standard trigonometric window $-6.28 \leq x \leq 6.28$, $-3 \leq y \leq 3$, the graph looks almost polynomial but with some strange double lines at the edge of the screen.

$-6.28 \leq x \leq 6.28$
$-3 \leq y \leq 3$



A trig friendly window would seem like the best bet ; however, by changing the window to a standard view $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, the nature of those "strange double lines becomes apparent as more of the curve and default asymptotes are revealed. Overlaying y = -2x can then be used to further describe the curve's behavior after soliciting observations from students.

$-10 \leq x \leq 10$
$-10 \leq y \leq 10$

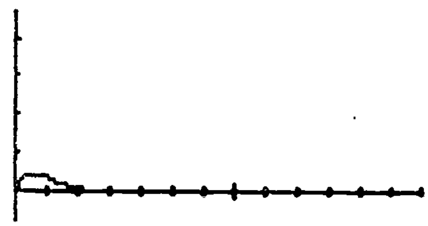


Example 10

Do math teachers stay up nights thinking of these tricky examples to torture us ?

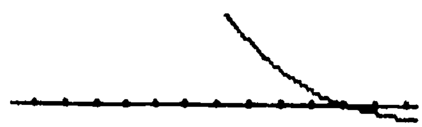
$$f(x) = e^{-3.5x}(1.55 \sin(x\sqrt{3.75}))$$

The given function is a variation (rounded constants) of the equation of motion in a critically damped system, a standard application of second order linear differential equations dealing with Hooke's Law and simple harmonic motion. In this situation, a slight decrease in damping results in oscillations. Graphing f(x) for $0 \leq x \leq 6$, $-1 \leq y \leq 3$, the curve seems to disappear into the x-axis . Where are the supposed oscillations?



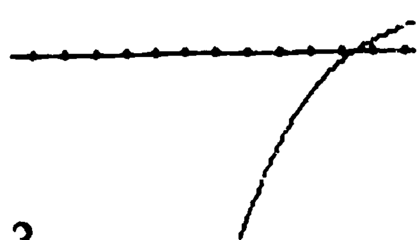
$0 \leq x \leq 6$
$-1 \leq y \leq 3$

Using the trace function , it is easy to see that the curve's y-values are both positive and negative ; however, most students will assume that the curve approaches the x-axis from above . The following view clearly shows the function crossing the x-axis from above . The y range is much smaller than most students anticipate.



$4.2 \leq x \leq 5$
$-1.0369E-7 \leq y \leq 6.1523E-8$

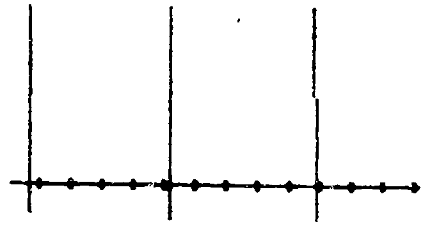
Again using the trace , it can be determined that the function will cross the x-axis next beyond 5.5 . The following graph shows the curve as it crosses the x-axis from below .



$5.8 \leq x \leq 6.6$
$-2.6509E-10 \leq y \leq 3.0632E-11$

An interesting view of the function uses a scale that exaggerates the curve's behavior into vertical lines . Observe the direction in which each segment is drawn.

$1.44 \leq x \leq 6$
$-3.8E-10 \leq y \leq 2.3E-9$

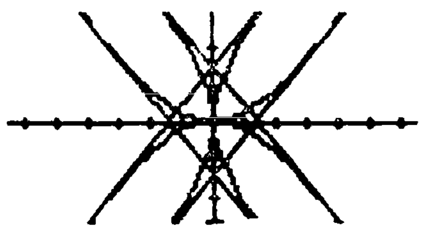


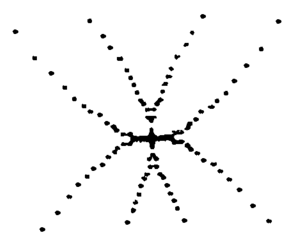
EXAMPLE 11
Can't we do a graph just for fun ?

$$r = 2\tan 2\theta$$

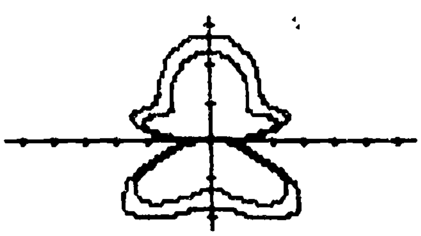
Viewed on a trigonometric window with $\theta_{min} = 0$ and $\theta_{max} = 6.28$, a surprisingly complex picture appears.

$-6.28 \leq x \leq 6.28$
$-3 \leq y \leq 3$





EXAMPLE 12
Can we do one more just for fun ?



$r_1 = \sin 5\theta - 5\sin\theta$
$r_2 = \sin 5\theta - 4\sin\theta$
$r_3 = \sin 5\theta + 5\sin\theta$
$r_4 = \sin 5\theta + 4\sin\theta$

$-7 \leq x \leq 7$
$-7 \leq y \leq 7$

$\theta_{min} = 0$
$\theta_{max} = 6.28$

dot mode
axes deleted

PRECALCULUS WORKSHEETS

This section contains original worksheets suitable for the precalculus level. Some materials are appropriate for use with any graphing calculator. Others are calculator specific (TI-81, HP48S) as indicated. These worksheets may be duplicated for non-commercial use .

SOLVING INEQUALITIES - A GRAPHING APPROACH

1. Solve $3x - 1 \leq 11$ and indicate your solution on a number line.

Graph $y = 3x - 1$ and $y = 11$ on the same set of axes. Find the intersection of the two curves.

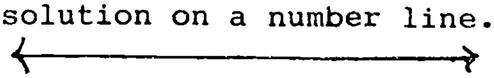
Explain how the xy-graph can be interpretted to indicate the solutions of the inequality.

2. Solve $3x - 1 \geq 11$ and indicate your solution on a number line.

Graph $y = 3x - 1$ and $y = 11$ on the same set of axes. Find the intersection of the two curves.

Explain how the xy-graph can be interpretted to indicate the solutions of the inequality.

3. Solve $2 \leq 3x - 1 \leq 11$ and indicate your solution on a number line.



Graph $y = 3x - 1$, $y = 2$ and $y = 11$ on the same axes. Find the intersection of the curves.

Explain how the xy-graph can be interpretted to indicate the solutions of the inequality.

4. Solve $|2x - 1| \leq 11$ and indicate your solution on a number line.

Graph $y = |2x - 1|$ and $y = 11$ on the same axes. Find the intersections of the curves .

Explain how the xy-graph helps justify that the solution is a continuous, closed interval. ______________________
____
____

Solve $|2x - 1| \geq 11$ and indicate your solution on a number line.

Explain how the xy-graph helps justify that the solution is two distinct intervals. ______________________
____
____

5. Solve $4 \leq |3x - 5| \leq 8$ both algebraically and graphically. Explain clearly the connection between the two approaches.

6. Give an example of a linear absolute value inequality which has no solution. Justify your claim with a graph.

7. Give an example of a linear absolute value inequality which has all real numbers as its solution. Justify your claim with a graph.

8. <u>Challenge</u> : Create quadratic absolute value inequalities which have solution sets consisting of one, two, and three closed intervals. Justify each with a graph.

NONLINEAR SYSTEMS OF EQUATIONS

I. Solve the following systems:(Review from your previous algebra courses.)

A) Use the method of elimination:

$$3x - 4y = 10$$
$$2x + 5y = -1$$

B) Use the method of substitution

$$4x + 3y = 10$$
$$3x - y = 1$$

II. Find the point(s) of intersection for:

a) State the type of graph each equation represents.
b) List the possible number of solutions.(i.e. 1,2,3 etc)
c) Solve the system using one of the methods in part I.
d) Set up the equations in the graphing utility and check your solutions.
   i) State your window.
   ii) Sketch the graph.

A) $$x^2 + y^2 = 16$$
$$x^2 - y^2 = 2$$

B) $$x^2 + 3y^2 = 7$$
$$x - y = 3$$

III. Set up the following system in your graphing utility and estimate the coordinates for the points of intersection with 3 decimal point accuracy in the x coordinate.

List the equations in the exact form you used in the graphing utility - complete with all the parenthesis.

$$x^2 - 3y^2 + 8x - 12y + 2 = 0$$

$$x^2 + 5y^2 - 16x + 10y - 4 = 0$$

COMPUTATIONS AND EQUATION SOLVING

The following problems require that you experiment with the hand held computer. Each problem will increase your awareness of fundamental facts of algebra. The conclusions you arrive at for each problem should be expressed carefully and submitted.

1. a) Use your calculator and test $x=-1$ and $x=3$ to determine whether or not they are solutions to the following equation.

   $$[(5x^2+1)/(2x^2+x-1)] - [x/(2x-1)] = (2x)/(x+1)$$

   b) Show how to use the multiplication and addition properties of equations to solve the above equation.

2. a) Use you calculator and test $x=-1$ and $x=5/6$, to determine or not those values of x are solutions to the following equation.

   $$|9x-2| = 3x-8$$

   b) Use the absolute value rule to find the solutions to the above equation.

3. a) Find a 5 decimal approximation for $(-1+\sqrt{13})/3$ and $(-1-\sqrt{13})/3$

   b) test each of you approximations to see if they are solutions the equation $3x^2+2x-4=0$

   c) Show how to use the quadratic formula or the method of completing the square to find the solutions to the equation in b .

4. a) Use your calculator to test whet er $x=8$ and $x=1/125$ are solutions to $5x^{2/3}-11x^{1/3}+2=0$

   b) Show how to find the solutions to the equation above by changing the equation to quadratic form.

5. a) Solve the inequality $x^2-7x+4 > 12$ by graphing $y=x^2-7x+4$ and $y=12$ on your calculator and using the trace function.

   b) solve the inequality above by using the properties of inequalities.

c) Solve the inequality $x^2 -7x -8 > 0$ on your calculator.

d) Explain why the results in a,b,and c are the same .

6. a) Solve the inequality $|7-9x| > 22$, by graphing the function $y=|7-9x|$ and $y = 22$ on the same cartesian plane.

b) Show how to solve the above inequality by using the properties of inequalities and the absolute value principle.

7. a) Solve the inequality $11/(x+3) < 2$ by graphing $y=11/(x+3)$ and $y=2$ on the same cartesian plane .

b) Solve the inequality in a by using the properties of inequalities

8. a) Use your calculator to solve the following equation for the variable x, by graphing $y=9x^{-2}-30x^{-1}+25$.

$$9x^{-2} -30x^{-1} +25=0$$

b) Solve the equation in a by changing the equation to quadratic form.

9. a) Solve the equation $-2x^3 +4x^2 -3x +6 =0$ by graphing.

$$y = -2x^3+4x^2 -3x +6$$

b) Solve the equation in a by using the properties of equations.

10. a) Solve the following equation by graphing

$$x^{2/3} -6x^{1/3} -16 =0$$

b) Solve the equation in by replacing $x^{1/3}$ by u and using the properties of equations.

HOW CONSTANTS CAUSE TRANSLATIONS, REFLECTIONS, AND COMPRESSIONS

CALCULATOR LAB

Given:

a. $y = a(x - b)^2 + c$

b. $y = a|x - b| + c$

c. $y = a\sqrt{x - b} + c$

Substitute numbers in for a, b, & c and graph the function. From these substitutions determine what a, b, & c do to the graph. Write a statement for each equation describing the effect a, b, & c have on the graph. Write your descriptions by comparing your graph with the graph where a = 1, b = 0, and c = 0.

What I expect to see in your report:

a) A graph for each function where a=1, b=0 and c=0. You may include additional graphs if you feel they are appropriate.

b) Descriptions for what changes in a, b and c do to the graph using properly structured paragraphs.

c) The modifiers it and they etc are removed from your paragraphs.

Due date for the project:

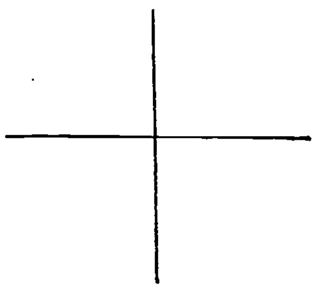
HORIZONTAL TRANSLATIONS OF $Y = LOG(X)$

1. Consider $y = \log_2 x$. Write it in exponential form . __________

2. Complete this table by computing values for x .

x	y
	-3
	-2
	-1
	0
	1
	2
	3

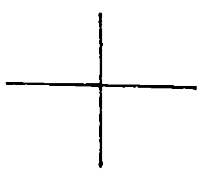
Sketch the graph.



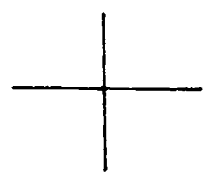
Domain : ______________
Range: ______________
Vertical Asymptote: ____

3. Using a graphing calculator , sketch each of the following.

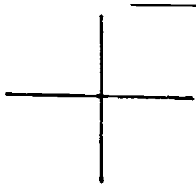
a) $y = \log(x + 2)$
Domain:__________



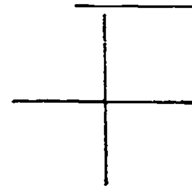
b) $y = \log(x + 1)$
Domain:__________



c) $y = \log(x - 1)$
Domain:__________



d) $y = \log(x - 2)$
Domain:__________



How can the domain of $y = \log(x \pm c)$ (where c is a constant) be found without drawing the graph ? ______________

____

____

Can you find the vertical asymptote from knowing the domain ? ______________________________

NON-QUADRATIC EQUATIONS SOLVED BY FACTORING

PROBLEM : Solve $x^6 - x^4 + 4x^2 - 4 = 0$

1. The function to be graphed is y = ______________________________

2. The solution(s) to the equation will be found on the graph where y = ____ . This is where the graph crosses the ____-axis .

3. Sketch the graph in the the standard viewing window.

5. How many solutions do you see ? ____
How many solutions did you expect ? ____
Can you guess why others do not show ?
____

6. Solve the equation by factoring . Show your work .

7. Now reconsider your answer to the last part of #5 .
____
____

8. Find a window which shows the complete graph of the function. Sketch the function.

Xmin = ____ Ymin = ____
Xmax = ____ Ymax = ____

GRAPHING PRACTICE

Directions : You are to answer the following questions by experimenting with your hand held computer. Make a careful sketch of the graph you see on the calculator for each question. Questions involving an explanation should have the answer written accurately in ink.

1. a) Graph $y = (x^2 + 4)^{\frac{1}{2}}$ and $y = -(x^2 + 4)^{\frac{1}{2}}$ on the same cartesian plane.

   b) Explain why the combined graphs of the two functions in a is the same as the graph of the equation $x^2+y^2=4$.

2. a) Graph $y = (25-x^2)^{\frac{1}{2}}$ and $y = -(25 - x^2)^{\frac{1}{2}}$ on the same cartesian plane.

   b) Explain why the combined graphs of the two functions in a is the same as the graph of the equation $x^2 + y^2 = 25$.

3. a) Graph $y = |x^2 + x - 12|$

   b) Graph $y = x^2 + x - 12$

   c) Graph $y = |2x + 1|$

   d) Graph $y = 2x + 1$

   e) Graph $y = (x^3 - 1)^{1/5}$.

   f) Graph $y = 1/(x + 4)$

4. a) Graph $y = x^2$, $y = (x-2)^2$, and $y = (x+2)^2$ on the same cartesian plane.

   b) Graph $y = (x+2)^2 + 2$, $y = (x+2)^2 - 2$, and $y = x^2$ on the same cartesian plane .

   c) Graph $y = -(x+2)^2$ and $y = x^2$ on the same cartesian plane.

   d) Give the coordinates of the vertices of each of the parabolas in a , b , and c . Also, give the vertex of the parabola $y=(x-h)^2 +k$.

5. a) Graph $y = (25 - (x-4)^2)^{1/2}$ and $y = -(25-(x-4)^2)^{1/2}$ on the same cartesian plane.

   b) Explain why the graph of $y^2 + (x-4)^2 = 25$ is the same as the combined graphs of part a .

6.a) Graph the following parametric set of equations. Use MODE PARAM setting. $x=t^2-3$
$y=t^2+4$

b) Solve each equation for t^2 and equate the two expressions to find a function of x. Graph the equation you found.

c) Why do the graphs in a and b differ ?

7. a) Graph the function $y = -2x^2 +20x -54$

b) Determine the maximum value of the function from the graph by using your computer.

c) Show how to find the maximum value of the function by using algebra.

8. Repeat the parts a,b,and c of problem 7 for the function $y= 3x^2 -12x +20$, except find the minimum value of the function.

9. Given that $f(x) = 1/(x-4)$ and $g(x) = 4x^2-8$, graph $f(g(x))$ and $g(f(x))$.

10. Use the methods for graphing piece wise functions to construct the graph of $f(x) = \begin{cases} x^2, & \text{if } x < -3 \\ x^2 +2, & \text{if } x > 0 \end{cases}$

Use the CALCULATOR to solve the following problems:

1. Consider the power function $p(x) = x^3$ and the exponential function $q(x) = 2^x$. Which one grows faster as x gets very large? Graph the two functions over the intervals:
   (a) -3 to 3
   (b) 0 to 10
   (c) 5 to 15
   Use the graph to determine to (two decimal places) where the graphs of these functions cross (intersect).

2. Which of the following is larger as x grows very large (i.e., as x approaches infinity),
   $f(x) = 100x^2$ or $g(x) = 0.1x^3$?
   Explain how you used the calculator to solve the problem. You may sketch the graphs you got from the calculator.

3. Graph the functions
   $F(x) = x^4$ and $G(x) = x^4 - 15x^2 - 15x$
   over the intervals
   (a) -20 to 20 (use y range -30 to 1000)
   (b) -4 to 4 (use y range -100 to 100)
   Describe what you notice about the graphs in both cases. What does this tell you about using the calculator?

EXPLORING INVERSE TRIGONOMETRIC FUNCTIONS (General)

Use a viewing window with X between −6.5 and 6.5, and Y between −2 and 2.

1. Graph Y = SIN (SIN^{-1}(X)). Is this what you expected to see? Why?

   Now erase and graph Y = SIN^{-1}(SIN(X)). Can you explain what you see?

2. Repeat this for Y = TAN (TAN^{-1}(X)) and Y = TAN^{-1} (TAN(X)). Whay are these different from the first pair?

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AN INTRODUCTION TO HORIZONTAL AND VERTICAL ASYMPTOTES AND LIMIT NOTATION

Vertical Asymptotes

Consider the function: $f(x)=\frac{1}{x}$. What is the name of this function? ____ What variable represents the elements in the domain of the function? ____ What represents the elements in the range of the function? ____ What is the domain of this function? On the x-y coordinate system, draw a dotted vertical line through the excluded point(s) on the x-axis. Investigate what happens to the values of the function as x gets closer and closer to the point(s) excluded from the domain of the function. To do this,

1. Choose values of x which are close to the excluded point, but to its left, and determine the corresponding values of f(x).

x	f(x)
− 0.1	
− 0.01	
− 0.001	
− 0.0001	
− 0.00001	
− 0.000001	
− 0.0000001	
− 0.00000001	

2. What happens to the values of the function f(x) as $x \to 0^-$? Please note: The notation "$x \to 0^-$" is read "x approaches zero from the left". It means that the numbers used to replace x in the rule of the function are to the left of zero, but each replacement is getting closer and closer to the number, zero.

____

____

3. Now, choose values of x which are close to the excluded point, but to its right, and determine the corresponding values of f(x).

x	f(x)
0.1	
0.01	
0.001	
0.0001	
0.00001	
0.000001	
0.0000001	
0.00000001	

4. What happens to the values of the function f(x) as $x \to 0^+$? Please note: The notation "$x \to 0^+$" is read "x approaches zero from the right". It means that the numbers used to replace x in the rule of the function are to the right of zero, but each replacement is getting closer and closer to the number, zero.

____

____

Definition: If $f(a)$ is not defined and $f(x) \to +\infty$ or $f(x) \to -\infty$ as $x \to a^-$ or $x \to a^+$, then the vertical line, $x = a$, is called a *vertical asymptote* of the function f(x).

Note: The notation "$f(x) \to +\infty$" *is read* "the value of the function approaches positive infinity" *and means the value of the function gets larger and larger as a positive number.*

Questions, before we continue:

1. How is "$f(x) \rightarrow -\infty$" read?

2. Interpret the meaning of "$f(x) \rightarrow +\infty$"

3. How is "$x \rightarrow +a$" read?

4. Interpret the meaning of "$x \rightarrow +a$".

5. How is "$x \rightarrow -a$" read?

6. Interpret the meaning of "$x \rightarrow -a$".

Horizontal Asymptotes

Once again, consider the function $f(x) = \frac{1}{x}$. However, this time let's examine what happens to the value of the function, $f(x)$, as x is replaced by positive numbers which get larger and larger. The notation used to describe this type of replacement for x is: $x \rightarrow +\infty$. The question to be answered is the following: "Does the value of the function, $f(x)$, approach some specific number as $x \rightarrow +\infty$? To answer this question, evaluate $f(x) = \frac{1}{x}$ for numbers which are positive and continue to get larger and larger in value.

x	f(x)
100	
1,000	
10,000	
100,000	
1,000,000	
10,000,000	
100,000,000	

What number do the values of the function f(x) seem to be getting closer to as $x \to +\infty$? ______ *Mathematically, we say that $f(x) \to 0$, as $x \to +\infty$.* The ***horizontal line***, *$y = 0$, is called a horizontal asymptote of the function, $f(x) = \frac{1}{x}$.*

Similarly, if the values of the function are examined as $x \to -\infty$, do the values of the function appear to get closer and closer to some specific number? To answer this question, evaluate $f(x) = \frac{1}{x}$ for numbers which are negative and continue to get more negative in value.

x	f(x)
− 100	
− 1,000	
− 10,000	
− 100,000	
− 1,000,000	
− 10,000,000	
− 100,000,000	

What number do the values of the function f(x) seem to be getting closer to as $x \to -\infty$? ______ *Mathematically, we say that $f(x) \to 0$, as $x \to -\infty$.*

Definition: If $f(x) \to a$, as $x \to -\infty$, then the horizontal line, $y = a$, is a *horizontal asymptote* for the function f(x). Similarly, If $f(x) \to b$, as $x \to +\infty$, then the horizontal line, $y = b$, is a *horizontal asymptote* for the function f(x).

Note: If a function has a horizontal asymptote, then for large positive or negative values of x, the graph of the function will get closer and closer to the horizontal asymptote, but the graph will never cross the asymptote or touch it.

HOMEWORK

Determine the domain, x-intercept(s), y-intercept, vertical and horizontal asymptotes of each of the following functions, and then draw their graphs. Use the calculator.

1. $f(x) = \frac{1}{x-1}$ 2. $f(x) = \frac{x}{x-1}$ 3. $f(x) = \frac{1}{x^2}$ 4. $f(x) = \frac{2x-4}{x-1}$

5. $f(x) = \frac{x^2}{x^2-4}$ 6. $f(x) = \frac{1}{x^2+9}$ 7. $f(x) = \frac{3x-6}{x^2-16}$ 8. $f(x) = \frac{x^2-1}{x^2+4}$

CHOOSING A GRAPHING WINDOW
USING THE TI-81 CALCULATOR

The screen of the TI-81 calculator is composed of tiny rectangular elements □ called *pixels*. There are ***95 pixels in each row*** of the screen and ***63 rows*** from the top to the bottom of the screen.

The length of the pixel determines the amount an x-location will increase or decrease as the cursor is moved to the right or left. For example, suppose the x-location of the cursor is $x = 4.3$ and the length of the pixel is 0.1. If the cursor is moved one unit to the right the x-location of the cursor becomes $x = 4.4$. If the x-location of the cursor is $x = 4.7$ and the pixel length is 0.5, what would be the new x-location of the cursor if it were moved 6 spaces to the left? (answer below)

Similarly, the height of the pixel determines the amount a y-location is increased or decreased as the cursor is moved up or down. If the height of the pixel is 0.3 and its y-location is $y = -1.8$, what is the new y-location if the cursor is moved down 2 spaces? (answer below)

Determining the Length of a Pixel

To determine the length of a pixel, consider the length of the screen. If Xmin and Xmax are assigned values, then

$$\text{length of screen} = \text{Xmax} - \text{Xmin} \qquad \text{(Eq. 1)}$$

Since 95 pixels make up the length of the screen, then the length of one pixel is found by dividing the length of the screen by the number of pixels.

$$\text{Pixel Length} = \frac{\text{Xmax} - \text{Xmin}}{95} \qquad \text{(Eq. 2)}$$

A more useful form of Eq. 2 is found by solving the equation for Xmax. The result is:

$$\text{Xmax} = \text{Xmin} + 95(\text{pixel length}) \qquad \text{(Eq. 3)}$$

Determining the Height of a Pixel

To determine the height of a pixel, consider the height of the screen. If Ymin and Ymax are assigned values, then

$$\text{height of the screen} = Y_{max} - Y_{min} \qquad (\text{Eq. } 4)$$

Since 63 pixels make up the height of the screen, then the height of one pixel is found by dividing the height of the screen by the number of pixels.

$$\text{pixel length} = \frac{Y_{max} - Y_{min}}{63} \qquad (\text{Eq. } 5)$$

A more useful form of Eq. 5 is found by solving the equation for Ymax. The result is:

$$Y_{max} = Y_{min} + 63(\text{pixel height}) \qquad (\text{Eq. } 6)$$

Graphing Piecewise Defined Functions on the TI-81

The TI-81 is capable of handling piecewise defined functions. For example:

$$f(x) = \begin{cases} 2 & x < 1 \\ x + 1 & x \geq 1 \end{cases}$$

The "pieces" can be stored in separate function locations ($y_1, y_2, \ldots$) and the domain indicated in parentheses as below:

$$y_1 = 2(x<1)$$
$$y_2 = (x+1)(x\geq 1)$$

OR

The "pieces" can be stored in one location as below:

$$y_1 = 2(x<1) + (x+1)(x\geq 1)$$

The inequalities are found in the TEST menu (2nd MATH).

A double inequality is entered as follows:
Example:

$$g(x) = \begin{cases} -2 & x < 2 \\ x^2 - 6 & 2 < x \leq 5 \\ x - 1 & x > 5 \end{cases}$$

As one function:

$$y_1 = -2(x<2) + (x^2-6)(2<x)(x\leq 5) + (x-1)(x>5)$$

OR

each stored in separate locations as before.

NOTE:

It is best to have the calculator in dot mode. (Change in the MODE menu.)

GRAPHING SPLIT DOMAIN FUNCTIONS (T1 81 WORKSHEET)

1) Set domain and range if desired (Range or Zoom)
2) Dot Mode
   MODE
   move cursor to DOT
   ENTER

GRAPH:

1. $f(x) = \begin{cases} x + 2, & x < 3 \\ x^2, & x \geq 3 \end{cases}$

y=
(
x
+
2
)
(
x
BLUE 2nd
MATH
5
3
)
+
(
x
x^2
)
(
x
BLUE 2nd
MATH
4
3
)
GRAPH

2) $f(x) = \begin{cases} -x, & x \leq -1 \\ 5, & -1 < x < 2 \\ 2x - 4, & x > 2 \end{cases}$

y=
CLEAR
(
(-)
x
)
(
x
BLUE 2nd
MATH
6
(-)
1
)
+
(
5
)
(
x
BLUE 2nd
MATH
3
(-)
1
)
(
x
BLUE 2nd
MATH
5
2
)
+
(
2
x
-
4
)
(
x
BLUE 2nd
MATH
3
2
)
GRAPH

TI 81 Worksheet

Evaluation of e

e is an irrational number. It is defined by $e = \lim_{x \to \infty}(1 + \frac{1}{x})^x$

The notation $\lim_{x \to \infty}$ means that you will let x become extremely large (or go off in the positive direction without bounds.

A. On your calculator put in

y=
(1 + (1÷x))^x
Blue 2nd Quit

1 STO x enter
Blue 2nd VARS ENTER ENTER

You should get 2. This is the value of $(1 + \frac{1}{x})^x$ when x = 1

Now do the same for the other values of x on the chart below.

x	Approximation of e
1	2
5	
10	
100	
1000	
10000	
100000	
1000000	
10000000	

Based on the above, what is a good approximation of e ?

B. now type in

Blue 2nd e^x 1 ENTER

Compare this answer with the one in part A

GRAPHING LINEAR AND QUADRATIC FUNCTIONS (TI-81)

Objectives: 1. To become familiar with the graphing keys

2. To examine solutions of equations by graphing
   a. linear functions
   b. quadratic functions

Example 1. Solve the equation $4x - 12 = 0$

Note $0 = 4x - 12$ is equivalent to

$y = 4x - 12$ when $y =$ _______

We will graph $y = 4x - 12$

Procedure: Push [y=] key and for y_1 type in 4 [X|T] - 12

What is the value of x when the function $y = 0$?

Push [Trace]

Push [▶] or [◀] and move the cursor along the graph until $y = 0$

Note y coordinate goes from - .63158 to .21053

To get exactly zero, we must change the range from the standard setting

Push [Range]

Set x min. = - 4.8
x max. = 4.7
y min. = - 3.2
y max. = 3.1

Push [Graph]

Push [y=] [<] [Enter] [∨]

Note the = sign is now unshaded and thus y_1 will NOT be graphed.

Assignment:

Plot the graphs of 1) $y - 3x - 1 = 0$
2) $y - 3x - 4 = 0$
3) $x = -3y + 6$

Remember to change to "y =" form.

Conclusions:

What can you say about lines 1) and 2) which have the same slope?

____

How do lines with a positive slope (1) and differ from lines with a negative slope (2)?

____

Suggest a window which would show the graph of $x = -3y + 9$ and enable you to trace with rational #'s as coordinates?

x min = ____________ y min = ____________

x max = ____________ y max = ____________

Example 2.

Using x as the variable, solve $x^2 - 2x = 35$

function to be graphed y = ____________________

Push $\boxed{y=}$ $\boxed{X|T}$ $\boxed{x^2}$ $- 2$ $\boxed{X|T}$ $- 35$ $\boxed{\text{Enter}}$

Graph

What can be done to see the whole parabola?

____

Use TRACE to find x when y coordinate is zero

Intercepts are x = ______________ and ______________

What is the relationship between the graph of $y = x^2 - 2x - 35$ and solution to the equation $x^2 - 2x = 35$?

____

Equal squares are cut off at each corner of a rectangular price of cardboard 3 ft. wide and 6 ft. long. A box is formed by turning up the sides.

(a) The volume of the box is ______________________________.

(b) Copy the graph of V(x) from your graphing calculator.

(c) Use the graph and your TRACE key to determine the approximate value of x which gives the maximum volume V(x).

EXPLORING POLYNOMIAL INEQUALITIES GRAPHICALLY (TI-81)

Objective: Exploring POLYNOMIAL INEQUALITIES graphically
Consider the function $y = 2x^2 + 3x - 5$
Graph on a friendly window (Note y min must be below -5)

1. Use $\boxed{\text{Trace}}$ to find the values of x when $y = 0$ ____________

2. What are the values of x when y is positive $\left(> 0 \right)$ ____________

3. What are the values of x when y is negative $\left(< 0 \right)$ ____________

4. Which set of values (those in 2 or 3) would be the solution to the inequality $2x^2 + 3x - 5 < 0$ ____________

5. Is this the same set of x's to solve $2x^2 + 3x < 5$? ____________

Exploring rational inequalities graphically

Consider the function $y = \frac{x - 1}{x + 2}$

Graph on a friendly window

1. Use $\boxed{\text{Trace}}$ to find the values of x when y changes its sign ____________

2. When x is in the interval $(-\infty\ -2)$ y is ____________

3. When x is in the interval $(-2, 1]$ y is ____________

4. When x is in the interval $[1, \infty)$ y is ____________

Therefore the solution to the inequality

$\frac{x - 1}{x + 2} \geq 0$ is ____________

GRAPHING CIRCLES (TI-81)

Objective: To graph equations of the form $x^2 + y^2 = r^2$

Example: Graph $x^2 + y^2 = 25$

The equation must be solved for y.

$$x^2 + y^2 = 25$$

$$y^2 = 25 - x^2$$

$$y = \pm \sqrt{25 - x^2}$$

Each expression must be graphed separately.

Push $\boxed{Y_1=}$ $\sqrt{(25 - x^2)}$ $\boxed{Y_2=}$ $-\sqrt{(25 - x^2)}$ $\boxed{\text{GRAPH}}$

Questions:

1. Did you get a circle? ____________________

2. If not, can you explain why? ______________

3. To get a perfectly round shape, we must ______
____

4. Will the entire circle $x^2 + y^2 = 121$ show in the default (standard) window?
____

5. If not, what window would you use?
____

6. What are the minimum window dimensions for viewing a complete circle $x^2 + y^2 = r^2$

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Exponential Functions Base e
(T181 Worksheet)

I. Graph $y = e^x$

A. It might be helpful to use different range for this work. Press the RANGE button (I assume that you are already in standard)

Move the cursor down to YMIN= set this to -1 ENTER
YMAX= set this to 15 ENTER

Blue 2nd QUIT

B. Y= (clear out anything previously located here)
Blue 2nd

e^x
x
GRAPH

C. What is the y intercept?

When you use the trace key, the cursor will miss x = 0 on this setting. There are two ways you can easily get the y intercept.

1. 0 STO X ENTER
Blue 2nd VARS ENTER ENTER

2. or set the RANGE Xmin to -9
Regraph and use the TRACE. This time you will stop on X = 0

D. Using the pattern 3 STO X ENTER BLUE 2nd VARS ENTER ENTER, complete the following table.

1.

x	e^x
3	20.0855
5	
10	
100	
-5	
-10	
-100	

Note 5.8 E 7 = 5 x 10^7

2. Based on the table and on the graph, what is the horizontal asymptote?

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II. Additional Examples

A. Graph

1. $y = e^{2x}$
2. $y = e^{4x}$
3. $y = 2e^{x}$
4. $y = 4e^{x}$
5. $y = e^{-x}$
6. $y = -e^{x}$

$\left.\begin{array}{l} 5. \\ 6. \end{array}\right\}$ You may need to set a different range

B. Compare each of the above with the graph of $y = e^{x}$

1. Consider the general shape
2. Consider the y intercept and the horizontal asymptote

C. Graph

1. $y = 2 - e^{x}$
2. $y = -4 + e^{x}$
3. $y = \dfrac{5}{1+2e^{-x}}$

Logarithms

I. Definitions $y = \text{Log}_B x$ if and only if $x = B^y$

or $x = \log_B y$ iff $y = B^x$

A. Examples

1. $2^3 = 8$ is the same as $\text{Log}_2(8) = 3$
2. $5^2 = 25$ is the same as $\text{Log}_5(25) = 2$
3. $4^{-1} = .25$ is the same as $\text{Log}_4(.25) = -1$
4. $\text{Log}_3(81) = 4$ is the same as $3^4 = 81$
5. $\text{Log}_{10}(.001) = -3$ is the same as $10^{-3} = .001$

B. Express the following in exponential form

1. $7^2 = 49$
2. $5^{-2} = .04$

C. Express the following in exponential form

1. $\text{Log}_2(64) = 6$
2. $\text{Log}_8(.002) = -3$

II. Two special types of Logs

A. Base 10

1. $\text{Log}_{10}(A)$ is called the common log of A. Its notation is Log(A) with no subscript.
2. Log (7) means $\text{Log}_{10}(7)$
3. On your calculator

   LOG 7 ENTER

   You should get .84509804

   This means $\text{Log}_{10}7 = .84509804$

   or that $10^{.84509804} = 7$

   BLUE 2nd 10^x BLUE ANS ENTER

B. Bace e

1. $\text{Log}_e(A)$ is called the natural log of A. Its notation is LN(A)
2. LN(7) means $\text{Log}_e(7)$

3. On your calculator

LN 7 ENTER

You should get 1.945910149

This means $Log_e(7) = 1.945910149$ or $e^{1.945910149} = 7$

BLUE 2nd e^x BLUE 2nd ANS ENTER

III. Graphing

A. Y_1: LOG x

Y_2: 10^xx [10^ x]

Y_3: x

Graph (Zoom Standard)

1. What do the graphs of $y = Log\ x$ and $y = 10^x$ suggest about these two finctions?

2. Try a) Log 10^x 5 ENTER

b) Log 10^x 132 ENTER

c) 10^x Log 120 ENTER

B. Y_1: LNX

Y_2: e^x

Y_3: x

Graph

1. What do the graphs of y= LNX and y= e^x suggest about these two functions?

2. Try a) LN e^x 7 ENTER

b) e^x LN 22 ENTER

USING THE TI-81 TO GRAPH A FUNCTION WHICH APPROXIMATES GIVEN POPULATION DATA

The data below is taken from "Population, Resources, Environment: An Uncertain Future," Robert Repetto, 1989, Population Reference Bureau Inc. (Note time is coded as 0, 50, . . . etc.)

The human population data is in billions (1×10^9)

time (calendar)	≈1E6 BC	1750	1800	1850	1900	1950	2000
time	1E6 BC	0	50	100	150	200	250
human population	0.0	0.8	1.0	1.2	1.7	2.8	6.2*

*projected population based on current trends.

Entering the data

Press [2nd] [STAT] [▶] [▶] [DATA]

Select [2] to clear any old data

Press [MODE] and select [DOT]

Press [2nd] [STAT] [DATA]

Select [1] and key in data . Set an appropriate range.

Investigating the function

Press [2nd] [DRAW]

Select [1] to clear any old graphs

Press [ENTER]

Press [2nd] [STAT] [▶] [DRAW]

Select [2] for scatter plot

We may connect the points to visualize the type of curve by pressing [2nd] [STAT] [DRAW]

Select [3]

What kind of curve does this resemble?

Determine whether a straight line or an exponential function fits better.

Press $\boxed{\text{2nd}}$ $\boxed{\text{STAT}}$ and select $\boxed{2}$

Note equation is $y = a + bx$
or $y = -.067 + .019x$
$r = .8587$

Copy this equation to the y-list

Press $\boxed{\text{Y=}}$

Press $\boxed{\text{VARS}}$ $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ $\boxed{\text{LR}}$

Select $\boxed{4}$

Linear function is copied to y_1

Press $\boxed{\text{2nd}}$ $\boxed{\text{STAT}}$ and select $\boxed{4}$

Note equation is $y = ab^x$
or $y = .657\ (1.008)^x$
$r = .9615$

Similarly copy the exponential equation to Y_2

Press $\boxed{\text{GRAPH}}$ and graph both equations

To see scatter plot, press $\boxed{\text{2nd}}$ $\boxed{\text{STAT}}$ $\boxed{\text{DRAW}}$ $\boxed{2}$

Visually, the exponential seems to be the better fit

analytically, r^2 measures the fit.

Compare .92 for the exponential to .74 for the linear function.

From the exponential function we can also assess the average rate of growth by examining b.

Recall growth curve $y = a(1 + \text{rate})^x$
compare with $y = .65(1 + .007)^x$
rate averages .7%

How long will it take the population to 8 billion?
Trace the exponential curve.

THE HYPERBOLIC FUNCTIONS - A DISCOVERY LESSON (TI 81)

1.a)Using the standard graphing window (ZOOM 6), graph y=sinh(x).

b)On the same axes, graph $y=x^3$.

c)Is y=sinh(x) the same as $y=x^3$?

1a)

d)Use ZOOM Box to investigate the relation between the two curves near the origin. Sketch on the given axes stating the range information of your box.

$x_{min}=$ ___ $x_{ma}=$ ___ $y_{min}=$ ___ $y_{max}=$ ___

e)Use TRACE to estimate the intersections to the nearest 100th. (,) (,) (,)

1d)

f)Complete this table of values for y=sinh(x).

x	y	x	y
-3		0	
-2		1	
-1		2	

g)Complete this table of values.

x_i from part e	y_i from part e	$\sinh(x_i) - (x_i)^3$

2a)

How good was your estimate in part e ?

2.a)After clearing all functions, draw the graph of y=cosh(x) using the standard viewing window (ZOOM 6).

b)Using your graph and your knowledge of exponents, guess each of these values:

$\lim_{x \to \infty} (\cosh x) = \lim_{x \to \infty} \frac{e^x + e^{-x}}{2}$ $\lim_{x \to -\infty} (\cosh x) = \lim_{x \to -\infty} \frac{e^x + e^{-x}}{2}$

c)Complete this table to confirm your response in part b.

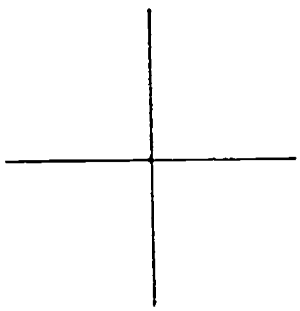
x	cosh(x)	$(e^{x})/2$	x	cosh(x)	$(e^{-x})/2$
50			-50		
100			-100		
150			-150		

d)Graph $y=\cosh(x)$, $y=(e^{x})/2$ and $y=(e^{-x})/2$ on the same axes.

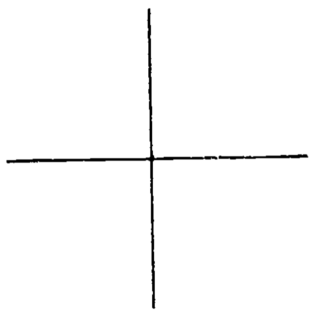
e)Use ZOOM BOX to investigate the relationship among these three curves.Sketch a graph to illustrate your findings.

3.Now use all you have learned to sketch the graphs of $y=\cosh(x)$, $y=\sinh(x)$, $(e^{x})/2$ and $(e^{-x})/2$ on the same axes.

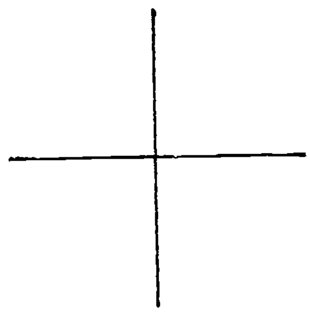
2d)



2e)



3)



INTRODUCTION TO PARAMETRIC EQUATIONS (TI-81)

You are used to finding points (x,y) on a graph by use of an equation involving x and y. For example one solution of $3x + 4y = 10$ is (2,1). There are many others. We plot these on a coordinate system and see a graph. Sometimes we use a calculator to do this.

Sometimes x and y depend upon a third variable, t, rather than directly upon each other. For example x could = 2t and y could = 3t. Then each time t takes on a value, x and y also take one a value. For example with t = 1, x=2 and y =3 giving the point (2,3). T=2 gives the point (4,6) and so on. Sometimes in applications, t represent time.

As another example. Let x = t and y = t^2 and consider the following table.

t	(x,y)
1	(1,1)
2	(2,4)
3	(3,9)
-1	(-1,1)

Equations such as the ones above are called parametric equations and t is called the parameter. Your calculator can graph parametric equations.

Select MODE
Use the down arrow to move the cursor to the fourth line down **Function** **Param**
Use the right arrow so that **Param** is flashing and not **Function**.
Select ENTER. Now **Param** should be hi-lited and not **Function**.
Select 2nd QUIT.

Your calculator will now operate in the parametric mode.

Select y= and you have different selections that when you were in function mode. You now must select functions for both x_1 and y_1. Let x_1 = T and y_1 = T^2 and above.
Choose ZOOM STANDARD and you will get a graph of half a parabola.
Using the arrows gives x and y coordinates of the points in the plane.
Using the trace and the arrows gives x and y coordiantes as well as values of T. Note that you cannot get to to be less than 0. Go to RANGE and change that, make TMIN and TMAX the same as the corresponding values of XMIN and XMAX respectively. Now GRAPH. Go back to RANGE and set TSTEP = 1. Then GRAPH. Why does this happen?

POLAR EQUATIONS ON THE TI-81

Polar equation to be graphed: $r = f(\theta)$

Polar coordinates conversion formulas : $x = r\cos\theta$ $y = r\sin\theta$
$= f(\theta)\cos\theta$ $= f(\theta)\sin\theta$

Graphing calculator T equations: $x = f(T)\cos T$ $y = f(T)\sin T$

EXAMPLE 1 (Begin this example with calculator in RECTANGULAR MODE.)

Graphing Calculator T Equations
$x = \sin 2T \cdot \cos T$
$y = \sin 2T \cdot \sin T$

Range:

a) Try ZOOM 6.
b) Try ZOOM 7.
c) Try
Tmin = 0
Tmax = 6.283185307] 0 to 2π
Tstep= .104719755
Xmin = -1
Xmax = 1
Xscl = .5 (Recall the range of sine and cosine is [-1,1])
Ymin = -1
Ymax = 1
Yscl = .5
The graph is out of proportion since the window is rectangular.
d) Try ZOOM 5 . Notice how values change for Xmin and Xmax.

Mode: Use GRID ON . Notice that range values remain.

Trace: Go to T = 3.5604717 X = -.6788966 Y = -.3022642

How do we find the ordered pair (r, θ) associated with a point on the parametric (T) graph ?
Check that you are in POLAR MODE. Use TRACE.
Go to T = 3.5604717 r = .74314483 θ = -2.722714
(Note: The calculator reports values using $r > 0$ and $-\pi < \theta < \pi$.)

Let's see the points that the TI-81 uses to draw the graph.
Use MODE: dot then TRACE. GO to T = 3.5604717 .

EXAMPLE 2

Graph $r = 1 - 2\cos\theta$. Use $x = (1 - 2\cos T)(\cos T)$
$y = (1 - 2\sin T)(\sin T)$

SIMULATING COIN TOSSES TO EXPLORE THE STATISTICS MENU (TI-81)

The following experiment is to be performed using the TI-81 graphing calculator. The experiment consists of simulating the toss of a fair die and recording the number of times the number two(2) occurs in six tosses of the die.

Keystroke Sequence:

Math ▶ [NUM] ▼ [IPart] ENTER

(6 * MATH ▶ ▶ ▶ [PRB] ENTER

) + 1 ENTER

If this sequence is performed correctly, the resulting display will be: IPart (6 * Rand) + 1. By depressing the ENTER key over and over, the calculator will display possible results of the toss of a die (simulate tossing a die). Count the number of times 2 occurs in six "tosses" of the die. Do this for 50 performances (simulations) of tossing a die six times. Keep track of the results in the following table:

X	TALLY	FREQUENCY
0		
1		
2		
3		
4		
5		
6		

When the simulation is complete, perform the following keystroke sequence:

2nd STAT (above MATRX) ▶ ▶ [DATA] ▼ [ClrStat] ENTER ENTER

2nd STAT ▶ ▶ [EDIT] ENTER

and enter your data as follows:

X1 = 0 ENTER
Y1 = (enter the frequency for 0) ENTER
X2 = 1 ENTER
Y2 = (enter the frequency for 1) ENTER

and so on, until all the data has been entered.

Then perform the keystroke sequence:

2nd STAT [1-Var] ENTER ENTER

and record the TI-81 display of:

$\bar{x}$ =	__________	Sx =	__________
Σx =	__________	σx =	__________
		n =	__________
Σx^2 =	__________		

Then perform the keystroke sequence:
2nd DRAW (above PRGM) ENTER ENTER
RANGE (top row just below the screen) and set the following:

Xmin = 0 ENTER
Xmax = 7 ENTER
Xscl = 1 ENTER
Ymin = 0 ENTER
Ymax = 25 ENTER
Yscl = 5 ENTER
Xres = 1 ENTER

2nd STAT ▶ ENTER ENTER

and the calculator will display a histogram for your data.

To turn off the calculator: 2nd OFF (above ON).

<u>EXPLORING THE VIEWING WINDOW (HP48S)</u>

1. **Viewing Window (Rectangle).** The display screen shows only a rectangular portion of the Coordinate Plane (x-y axes). This is determined by XRNG (x range) and YRNG (y range). The standard window is XRNG -6.5 to 6.5 and YRNG -3.1 to 3.2. WHY? We'll see shortly. This gives a rectangle of [-6.5,6.5] by [-3.1,3.2].

The display screen consists of a rectangular array of "lights" called **pixels**. A point on the screen is shown by lighting up one of the pixels.

PROBLEM 1. Set XRNG to -10 to 10 and YRNG to -10 to 10. Go to the PLOT menu then PLOTR and graph anything. Now press COORD to see where the **cursor** is located, and use the arrow keys to move the **cursor** left, right, up, and down. Observe the coordinates as you proceed.

(a) What are the screen coordinates of the four corners of the display screen?

(b) Now change the viewing window by going back to PLOTR and using RESET (2nd page of the menu). Move the cursor around again, examining the coordinates as you proceed. What is the difference in coordinate value types? Are the values easier to read? WHY?

The display screen has 131 columns and 64 rows of pixels, for a total of ______ pixels.

PROBLEM 2. Graph the function

$$y = -3x^2 + 12x + 5$$

by using, in turn, the following viewing rectangles:

(a) [0,5] by [0,5] (that is, XRNG 0 to 5, YRNG 0 to 5)

(b) [-10,10] by [-10,10]

(c) [-5,20] by [-10,20] (XRNG -5 to ___, YRNG ___ to ___)

Which of these gives a "complete" graph?

Now focus in on the "flat" portion of the graph by viewing the graph in [0,1.5] by [-30.2,-29.6] or by zooming in after moving the cursor to the flat portion.

PROBLEM 3. Find a good viewing rectangle for a "complete" graph of

(a) $y = 2x^2 - 40x + 150$

(b) $y = 20 + 9x - x^3$

EXPLORING ABSOLUTE VALUE AND STEP FUNCTIONS (HP48S)

SPECIAL FUNCTIONS

This lesson will help you to graph and explore several special functions in Math, namely the ABSOLUTE VALUE function, the FLOOR function (greatest integer) and the CEILING function. These can all be found in the MTH menu (R2,C1) under PARTS.

1.(a) Press the MTH key, followed by PARTS (first white key)
'ABS α X ENTER ==> to get ABS(X) on the stack
' Y STO ==> to store it into Y
Now graph using a XRNG of -4 to 4 and AUTO under PLOT

(b) Using VAR, recall the value Y ('Y ENTER) and EDIT the equation to get ABS(X - 4) then graph **without** erasing. How does this compare to (a).

2.(a) FLOOR is defined as FLOOR(X) = largest integer which is less than or equal to X, e.g., FLOOR(5.3) = 5, FLOOR(0.7) = 0.
Press MTH, then PARTS. FLOOR is on the third page of the PARTS menu.
'FLOOR α X ENTER ==> to get FLOOR(X) on stack
' α Y STO ==> to store it into Y
Now graph, using an XRNG of -4 to 4 again. What does the graph appear to look like?
To get a better idea of the "correct" version, use ORANGE MODES (R2, C3) go to page 2 of the menu and press the white key for CNC to remove the white square, turning off CONNECT. Then replot the function. Now what do you see?

(b) Now go back and plot FLOOR(X + 2). How does this compare to (a)?

3. (a) CEILing(X) = smallest integer >= X.
CEILing(4.3) = 5 and CEILing(-3.2) = -3.

CEIL can be found next to FLOOR in the MTH PARTS menu.
Graph Y = CEIL(X) and CEIL(2*X) and CEIL(X -1).

EXPLORING PIECE-WISE FUNCTIONS (HP48S)

FUNCTIONS DEFINED IN PIECES (Piecewise)

This lesson will help you in using the graphing calculator to graph functions defined in two or more "pieces". First make sure you are in CNC mode by checking MODES.

EXAMPLE:

$$f(x) = \begin{cases} -2x \text{ if } x < 0 \\ x^3 \text{ if } x >= 0 \end{cases}$$

This function really says,

```
  given a value x:                     +------------------+
    if x < 0 then                      |  if x < 0 then   |
      compute -2*x as f(x)     x -->   |    y <-- -2*x    |  --> y
    otherwise (else x >= 0)            |  else            |
      compute x^3 as f(x)              |    y <-- x^3     |
                                       +------------------+
```

The calculator uses programming (PRG) features to define the function:

Press PRG (R2,C2) and go to the BRCH (branch) menu; then NEXT NEXT to get to page 3 of that menu to find IFTE (if-then-else). IFTE has the form

IFTE(condition, expression, expression) so we press

'IFTE ===> gives IFTE( )

X PRG TEST (menu) NEXT (page 2) < 0 ORANGE , -2 * X ORANGE , X y^x 3 ENTER

' Y STO ===> to store the function in Y

Now go to the plot menu to plot this function in the usual way; find a good viewing window.

EXAMPLE: Now graph the function

$$y = f(x) = \begin{cases} 2x - 1 \text{ if } x <= -2 \\ x^2 \quad\quad \text{ if } x > -2 \end{cases}$$

What does the graph appear to show?

Go to the MODES menu and change CNC if necessary.

EXAMPLE: How would we define, using the calculator

$$y = f(x) = \begin{cases} 2*x & \text{if } x < -2 \\ -4 & \text{if } -2 <= x < 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

To accomplish this, we must replace the second expression in IFTE by another IFTE, as follows:

' IFTE(X <= -2, 2*X, IFTE(X < 2, -4, X - 3 ENTER

' α Y STO

Now plot this function.

EXPLORING PROPERTIES OF GRAPHS (HP48S)

1. Graph $y = x\sqrt{5 - x^2}$ after RESETting window.
   Domain = ?
   Range = ?
   Try to find an XRNG which gives a "complete" graph, that is, one which reaches the x-axis at its ends.
   Use the FCN menu to determine the (approximate) high and low points on the graph. This is done by moving the crosshair close to the desired point and pressing the EXTR white key.
   Low point = (,   ), High point = (,   ).

2. Graph $y = x^4 - x^2$ on XRNG [-2,2] and YRNG [-1,1]
   $y = x^6 - x^4$
   $y = x^3 - x$
   $y = x^5 - x^3$

   Discuss the similarities and differences in the graphs, including ranges, intercepts, where the graphs are increasing and decreasing, and high and low points. You may graph them together and/or separately.

   Also graph $x^4 - 1$ on the same grid with a wider XRNG of -6 to 6. Compare the graphs close to 0 and away from 0.

3. Enter the equation $2x + 1 = (x - 1)/2$ and graph it using an XRNG of [-3,3] and YRNG of [-2,2]. What do you notice about these graphs? Are they inverses? How can you tell. Is the first one-to-one? The second? Draw $y = x$ on the same set of axes.

   Enter the equation $x^3 = \sqrt[3]{x}$ in the window [-3,3] for X, [-2,2] for Y. Then superimpose $y = x$ on this graph. What do you notice about the graph of $y = x$ and the other two graphs?

EXPLORING PARABOLAS (HP48S)

Using the EQUATIONwriter:

The EQUATIONwriter allows us to enter expressions and equations in a simpler fashion. This lesson will illustrate the idea by entering the equation

$$y - k = a(x - h)^2$$

and graphing for different sets of values for the parameters a, h, k.

1. PURGE X and Y:
   ' α X ORANGE PURGE
   ' α Y ORANGE PURGE
2. Use the equationwriter:
   ORANGE EQUATION (ENTER key)
   α Y - α K ORANGE = (0 key)
   α A * ORANGE () α X - α H > y^x 2 > ENTER
   to get the equation onto the stack.
   ' PARAB STO
3. Now select VAR to see the defined equations & functions.

EXPLORING POLYNOMIAL FUNCTIONS (HP48S)

In this session you will examine the behavior of polynomial functions at the "ends", far from the origin, and close to the "middle" of the functions. The choice of an XRNG and YRNG over which to graph will make a big difference in the way you view the graph. You should experiment to get the "best" possible view.

To see what is meant by the above remark, first graph $y = x^4 - 15x^2 + 2$ over an XRNG [-20,20] using AUTO. Before graphing, discuss the symmetry of the graph. Discuss the features of this graph. What graph does it resemble? Now graph $y = x^4$ on the same set of axes. What do you observe? Does this give the total picture of the function? Now go back and graph the original function over an XRNG of [-4,4] using AUTO. Now what can you say about the graph? Notice the difference between the behavior close to the origin and far away from the origin.

CUBICS

1. Recall the shape of the simplest cubic polynomial, $y = x^3$. You should already know what happens to that shape if you make slight changes, such as $y = -3x^3$ or $y = 2(x + 3)^3$. Make a quick sketch of those three graphs, without using your calculator. How many x-intercepts does each one have? Show these carefully.

2. (a) Now try graphing the following cubic:
$y = f(x) = x^3 - 6x^2 + 8x$
on XRNG [-10,10] AUTO. How many x-intercepts? What are they? If you can't tell draw the graph using XRNG [-1,5]. Now what are the x-intercepts? Use FCN (from the menu) and find the x-intercepts and high and low points (turning points) using ROOT and EXTR from the FCN menu. Label these locations carefully.
(b) Try the same thing on $f(x) = x^3 - 6x^2 - 11x + 6$. Use a narrow interval on the x-axis. Sketch the graph labeling the x-intercepts and high and low points (turning points). Does the graph resemble $y = x^3$ at all? How. How do they differ?

3. Clear the screen and try two more graphs, one at a time:
(a) $y = g(x) = x^3 - 5x^2 + 8x - 4$
(b) $y = h(x) = x^3 + x^2 + x - 3$
Note (i) the number of roots (zeros, x-intercepts)
(ii) the number of turning points
(iii) whether the ends of the graph go up or down.
Write your results.

4. Now graph the following cubics, one at a time. Observe the similarities and differences as well as the features discussed above. Write your observations:
(a) $y = -x^3 - 6x^2 + 11x - 6$
(b) $y = -x^3 - 5x^2 + 8x + 4$
(c) $y = -x^3 + x^2 + 3x + 4$
(d) $y = -x^3 - x^2 - 3x + 4$

5. Try some cubics with leading coefficients not equal to 1 or -1. Now try to generalize about the graphs of cubic polynomials.

Graphing Calculator Explorations (HP48S)

1. Find the simultaneous solution to the system of equations

$y = x^2$

$y = 2^x$

That is find where $x^2 = 2^x$. To do this, first graph the set of equations $\{x^2 \quad 2^x\}$ in the standard viewing rectangle. Where is $2^x > x^2$ and where is $2^x < x^2$?

RECALL that you can graph several equations at one time by using ORANGE {} ' EQ1' SPC 'EQ2' ..

2. Draw complete graphs of the following functions, examining the end behavior of the functions as x —> + oo:

(a) $f(x) = \dfrac{x^2}{\ln x}$

(b) $g(x) = \dfrac{\ln x}{x}$

(c) $h(x) = \dfrac{x}{\ln x^2}$ (examine this also as x —> 1^-

3. The area of a triangle is to be 150 sq. ft. Draw a triangle.

(a) Write the height of the triangle as a function of the length x of its base. Draw a complete graph of this function.

(b) Find the domain and range (graphically). What values of x make sense in the context of the problem, that is, in relation to what x represents?

(c) If height = 800, find x.

4. Graph the following systems of equations on the same coordinate system and find the points of intersection by moving the cursor and/or using ISECT under the FCN menu. FIRST solve each equation for y:

(a) $x^2 - y = 0$
$y - 2x - 3 = 0$

(b) $x + 3y = -1$
$2x - y = 5$

(c) $3x + y = 6$
$6x + 2y = 12$

(d) $x^2 + y = 19$
$x^2 + y^2 = 25$

Rational Function Explorations (HP48S)

A rational function is a function of the form $\frac{P(x)}{Q(x)}$ where P and Q are polynomials. Your assignment, in preparation for a general discussion of rational functions, is to

1) graph the given functions, first in the standard window (use RESET if necessary);

2) describe features of the graphs - where they are increasing, decreasing, any turning points, the end behavior.

First graph each function in the standard window. Then change XRNG to 0 to 13, then 10 to 23, then 20 to 33 to explore the right end behavior. Adjust YRNG if necessary, or use AUTO. Similarly, to explore the left end behavior (negative x), use -13 to 0, then -23 to -10, then -33 to -20.

NOTE: These XRNGs are to keep the difference between the lower and upper ends equal to 13, the same as -6.5 to 6.5 (standard).

Function set 1: (a) $\frac{1}{x}$ (b) $\frac{1}{x - 1}$ (c) $\frac{x}{x - 1}$ (d) $\frac{x^2}{x - 1}$

Function set 2: (a) $\frac{1}{x + 3}$ (b) $\frac{x}{x + 3}$ (c) $\frac{x^2}{x + 3}$

Function set 3: (a) $\frac{1}{x^2}$ (b) $\frac{1}{x^2 - 4}$ (c) $\frac{x}{x^2 - 4}$ (d) $\frac{x^2}{x^2 - 4}$

Function set 4: (a) $\frac{1}{x^2 + 4}$ (b) $\frac{x}{x^2 + 4}$ (c) $\frac{x^2}{x^2 + 4}$

Draw rough sketches of the curves, copying the viewing screen of your calculator. Label key points. Discuss the features listed above, using the calculator as an aid. Discuss any similarities and differences within each function set and between function sets.

EXPLORING POLAR GRAPHS USING THE HP-48S

Be sure to set the PTYPE to POLAR
XRNG to -1 1
YRNG to -1 1
INDEP to {'T' 0 6.29}

1. Graph the polar equations R= SIN(2*T);
R= COS(2*T);
R = SIN(3*T);
R= COS(3*T).

Explain the similarities and differences you see in the four graphs.

2. Now graph R = COS(1.35*T);
R = COS(1.4*T);
R = COS(1.5*T).

Explain what happened in each graph. Why are they similar? Different?

3. Graph R =SIN(2.5*T);
R =SIN(2.8*T);
R =SIN(3.1*T);
R =SIN(3.25*T);
R =SIN(7.35*T).

Explain what happened in each of these graphs. Why are they similar? Different?

4. Considering all the graphs you just did, can you state a general property for these rose curves?

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CALCULUS WORKSHEETS

This section contains original worksheets suitable for the calculus level. Some materials are appropriate for use with any graphing calculator. Others are calculator specific (TI-81, HP48S) as indicated. These worksheets may be duplicated for non-commercial use .

USING NEWTON'S METHOD TO FIND ROOTS OF A POLYNOMIAL (General)

Newton's Method is a process for approximating the roots of a function and utilizes the following procedure. Perform the actual process from step 1 through step 4 for the function given by $y = x^3 - x + 1$ using $x = -1$ as your first guess.

1) Guess at a root of the equation. 1. $x =$ ______

2) Find the coordinates of the point on the curve. 2. $y =$ ______

3) Find the equation of the tangent line to the curve. 3. $y =$ ______

4) Find the x–value where the tangent line intersects the x–axis. This is the next approximation and would be used in step 1. 4. $x =$ ______

5) Use your calculator to sketch a graph of the function and and the tangent line.

6) The process now repeats and is summarized in the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

7) Use this formula to find a root of f(x) to ten decimal places. Place your successive x values in the table below. Add to it if necessary.

x_1 −1

x_2 ____________

x_3 ____________

x_4 ____________

x_5 ____________

x_6 ____________

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OBJECTIVE----The objective of this exercise is to use the trace feature to estimate limits of functions at specified values.

CLASS EXAMPLES

1. Use $f(x) = \frac{|x|}{x}$ to complete the following.

Graph f(x) and use trace to complete the table below for f(x) as x approaches 0 from both the left and right. Use $-2.4 \le x \le 2.35$ and $-3.2 \le y \le 3.1$

x	-.2	-.15	-.1	-.05	0	.05	.1	.15	.2
f(x)									

1) $\lim_{x \to 0^-} f(x) =$

2) $\lim_{x \to 0^+} f(x) =$

3) $\lim_{x \to 0} f(x) =$

4) $f(0) =$

2. Use $f(x) = \frac{x}{x+2}$ to complete the following.

Graph f(x) and use trace to complete the table below for f(x) as x approaches -2 from both the left and right. Use $-3.4 \le x \le 1.35$ and $-20 \le y \le 20$

x	-2.2	-2.15	-2.1	-2.05	-2	-1.95	-1.9	-1.85	-1.8
f(x)									

1) $\lim_{x \to -2^-} f(x) =$

2) $\lim_{x \to -2^+} f(x) =$

3) $\lim_{x \to -2} f(x) =$

4) $f(-2) =$

3. Use $f(x) = \frac{x^2-1}{x-1}$ to complete the following. Create your own table-of-values and graph the function in the window $-4.8 \le x \le 4.7$ and $-3.2 \le y \le 3.1$.

1) $\lim_{x \to 1^-} f(x) =$

2) $\lim_{x \to 1^+} f(x) =$

3) $\lim_{x \to 1} f(x) =$

4) $f(1) =$

HOMEWORK EXERCISES

1. $f(x) = \dfrac{5}{2x-1}$ $\quad -4.8 \le x \le 4.7$ and $-32 \le y \le 31$

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
f(x)									

1) $\lim_{x \to \frac{1}{2}^-} f(x) =$

2) $\lim_{x \to \frac{1}{2}^+} f(x) =$

3) $\lim_{x \to \frac{1}{2}} f(x) =$

4) $f(\frac{1}{2}) =$

2. $f(x) = \dfrac{\sin x}{x}$ $\quad -4.8 \le x \le 4.7$ and $-3.2 \le y \le 3.1$

x	−.4	−.3	−.2	−.1	0	.1	.2	.3	.4
f(x)									

1) $\lim_{x \to 0^-} f(x) =$

2) $\lim_{x \to 0^+} f(x) =$

3) $\lim_{x \to 0} f(x) =$

4) $f(0) =$

3. $f(x) = \dfrac{x^2 - 4}{4x + 8}$ $\quad -3.4 \le x \le 1.35$ and $-3.2 \le y \le 3.1$

x	−2.2	−2.15	−2.1	−2.05	−2	−1.95	−1.9	−1.85	−1.8
f(x)									

1) $\lim_{x \to -2^-} f(x) =$

2) $\lim_{x \to -2^+} f(x) =$

3) $\lim_{x \to -2} f(x) =$

4) $f(-2) =$

OBJECTIVE----The objective of this exercise is to use the trace and statistics features to graphically estimate the numbers, c, that satisfy the conclusion of the Mean Value Theorem.

CLASS EXAMPLE

Step 1: Set $y_1 = 5x^2 - 3x + 1$.

Step 2: Press $\boxed{2^{nd}}$ $\boxed{STAT}$.
Move the cursor to 'DATA' and press $\boxed{ENTER}$.

Step 3: Set $x_1 = 0$, $y_1 = 1$ and $x_2 = 2$, $y_2 = 15$. These are the two points on graph y_1 that we are using to create our secant line. Therefore the interval over which we are analyzing is [0,2] .

Step 4: Press $\boxed{2^{nd}}$ $\boxed{STAT}$.
Move the cursor to 'LinReg' and press $\boxed{ENTER}$.
Press $\boxed{ENTER}$ again.
Press y= and move the cursor to y_2. Press $\boxed{VARS}$.
Move the cursor to 'LR'.
Highlight 'RegEQ' and press $\boxed{ENTER}$.
What did this step create in y_2= ? ____________________

Set $y_3 = y_2 - y_1$.

Step 5: Using range factors of xmin = -.25, xmax = 2.125, ymin = -5, ymax = 20, graph y_1 and y_2 on the same grid. We are seeing the graph of the function and the graph of the secant line.

Step 6: Clear the graphs of y_1 and y_2. Graph y_3.
Use the trace key to find the max or min value(s) of y over the interval being analyzed. The corresponding x-value(s) estimate the c for the Mean Value Theorem.
c =

HOMEWORK EXERCISES

1. $y_1 = 2.1x^4 - 1.4x^3 + 0.8x^2 - 1$

points (0,-1) and $(1,\frac{1}{2})$

xmin = -1
xmax = 2
ymin = -2
ymax = 2

answer: $c \approx .64$

2. $y_1 = \dfrac{\sin(2x) + \cos x}{2 + \cos(\pi x)}$

points (0,1/3) and (1,1.4495994)

xmin = -.25
xmax = 1.25
ymin = -.5
ymax = 2

answers: $c \approx .30$, .84

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INVESTIGATING SLOPE AND THE NDeriv FUNCTION ON THE TI-81

OBJECTIVE----The objective of this exercise is to become familiar with the use of the NDeriv key and how it relates to the slope of the secant line.

Following is the description of the NDeriv key from the TI-81 calculator manual.

> NDeriv(requires two arguments. The first argument is an expression in terms of X. The second argument is a delta X. The numerical derivative value is the slope of the secant line through the points $(X-\Delta X, f(X-\Delta X))$ and $(X+\Delta X, f(X+\Delta X))$ for the current value of X. This is an approximation of the numerical derivative of the function at X. As ΔX gets smaller, the approximation usually gets more accurate.

CLASS EXAMPLES

For the given function, estimate the desired derivative using $\Delta x = .001$.

1. $f(x) = x^2 - 3x + 4$ $\quad f'(2) =$

2. $f(x) = 3x^3 - 2x$ $\quad f'(1) =$

3. $f(x) = x^4 + x^3 - x^2 - 2$ $\quad f'(-2) =$

4. $f(x) = \sec x$ $\quad f'(\frac{\pi}{4}) =$

HOMEWORK EXERCISES

1. $f(x) = 2x^2 - x + 3$ $\quad f'(3) =$

2. $f(x) = x^3 + 2x^2 - x$ $\quad f'(-1) =$

3. $f(x) = -3x^4 + x^2 - 1$ $\quad f'(\frac{3}{2}) =$

4. $f(x) = 2 \cos 3x$ $\quad f'(-3\pi/2) =$

5. $f(x) = \tan x + \sin x$ $\quad f'(\pi) =$

GRAPHING THE DERIVATIVE OF A FUNCTION USING THE TI-81

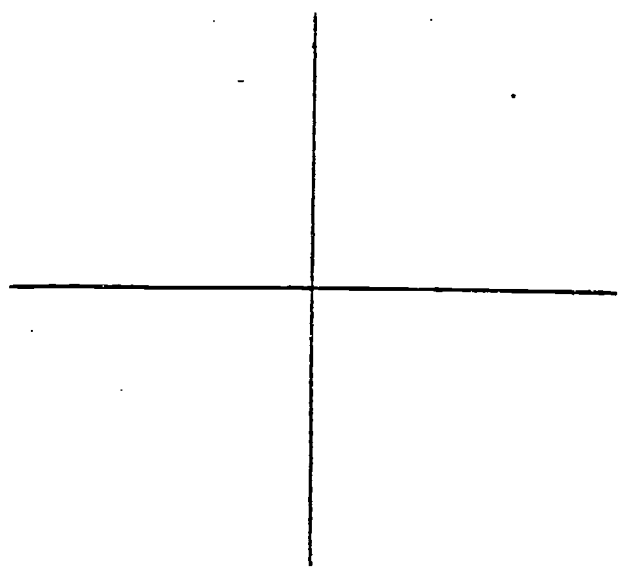
JECTIVE----The objective of this exercise is to use the NDeriv key to graph the derivative of a function.

LASS EXAMPLES

For the following functions, a) enter f in y_1 and graph f over the given domain and range, b) enter NDeriv(y_1,.0001) in y_2 and graph y_2, c) determine the equation for f' and enter it in the space below, and d) draw and label both graphs on the grid provided.

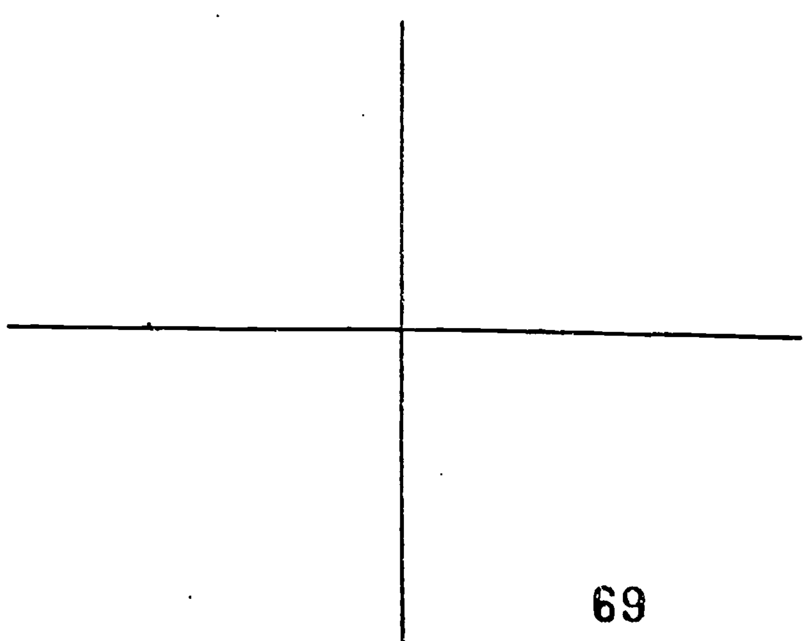
1. $f(x) = x^2 - 3x + 4$; standard values

   $f'(x) =$



2. $f(x) = 2\cos x$; $-6.28 \le x \le 6.28$ and $-3 \le y \le 3$

   $f'(x) =$



1. Let $f(x) = \frac{\sin(4x)}{\sin(2x)}$ and $g(x) = \frac{\sin(5x)}{\sin(2x)}$.

a) Use the standard viewing rectangle to graph f(x). Sketch the graph below. The standard viewing rectangle is given by $-6.5 < x < 6.5$ and $-3.1 < y < 3.2$.

b) From your graph determine each of the following limits.

$\lim_{x \to \frac{\pi}{2}} f(x) =$ ____________ $\lim_{x \to 0} f(x) =$ ____________

c) Use the standard viewing rectangle to graph g(x). Sketch the graph here.

d) From your graph determine each of the following limits.

$\lim_{x \to \frac{\pi}{2}} g(x) =$ ____________ $\lim_{x \to 0} g(x) =$ ____________

e) Complete the following tables to observe some of the above results numerically.

x	−.1	−.01	−.001	.001	.01	.1
g(x)						

What is $\lim_{x \to 0^-} g(x)$? ____________ What is $\lim_{x \to 0^+} g(x)$? ____________

x	1.56	1.565	1.57	1.575	1.58
g(x)					

What is $\lim_{x \to \frac{\pi}{2}^-} g(x)$? ____________ What is $\lim_{x \to \frac{\pi}{2}^+} g(x)$? ____________

2. Let $h(x) = \frac{\sin(\pi x)}{|x-1|}$. Use a viewing rectangle $-2 < x < 4$ and $-4 < y < 4$ and graph $h(x)$.

a) Sketch your graph here.

b) From your graph determine each of the following limits.

$\lim_{x \to 1^-} h(x) =$ ________________ $\lim_{x \to 1^+} h(x) =$ ________

c) Complete the following table to observe some of the above results.

x	.99	.999	1.001	1.01
g(x)				

What is $\lim_{x \to 1^-} h(x)$?________ What is $\lim_{x \to 1^+} h(x)$? ______

1) Graph sinx in the SVR.
   On your graph determine each of the following function values.

   f(.7) = __________ f(.8) = ____________

   Use this information to compute the slope of the line through those points. (m =) _____

   Use symbolic methods to compute f′(.75) when f(x) = sinx. Show your work.

   Find the equation of the line tangent to sinx at your computed value (.7, f(.7)). (Show your work below.) Draw the line on your calculator, but do not erase your previous graph. You will now have a graph of sinx and a line tangent to it.

2. a) Graph $y = x^2$ in the SVR. Leave the cursor at the origin. Zoom in using an xy factor of .01. Sketch a picture of the graph, particularly the way it looks around (0,0).

   b) Graph $y = |x|$ in the SVR. Leave the cursor at the origin. Zoom in using an xy factor of .01. Sketch a picture of the graph, particularly the way it looks around (0,0).

   c) Compare your two graphs in a and b noting the difference between a graph where the derivative exists and where one does not.

USING DERIVATIVES & DIFFERENTIALS TO APPROXIMATE FUNCTION VALUES USING THE HP-48S

1) Consider the graph of the function $f(x) = x^3 - 9x + 11$. Find an equation of the line tangent to the curve at the point (2,1).

2) Enter the following into level 1 of your calculator.
{'x^3 - 9*x+11' '3*x-5'} Treat this as a function and graph it in the SVR. Sketch the graph below. This should be the graph of a function and a line tangent to it.

3) Center the graphs at (2,1) and zoom in with a factor of .2. Move the cursor around to approximate where the line and the curve appear to have the same coordinates.
What is the minimum x-coordinate?__________

What is the maximum x-coordinate?__________

4) Complete the following table using the x-coordinates you obtained above to calculate the corresponding y-coordinates.

	$x^3 - 9x + 11$	$3x - 5$
x_{min}		
x_{max}		

5) Both curves go through the point (2,1). Sketch a graph that emphasizes the differences illustrated by the above numbers. Label each point with its coordinates.

6) Recall $\Delta y = f(x_0+\Delta x) - f(x_0)$. Calculate this difference using the point (2,1) and $\Delta x = x_{max} - 2$for the cubic function above.

7) dy is the corresponding differences measured from the tangent line rather than the cubic. Calculate this value using the points (2,1) and $(x_{max}, f(x_{max}))$.

8) Verify that $dy = f'(x_0)dx$ where $dx = \Delta x$ by calculating the product $f'(x_0)dx$ at $x_0 = 2$.

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Recall that the linearization of a function f(x) at x = a is

$$L(x) = f(a) + f'(a)*(x - a),$$

that is, the tangent line at x = a. This line is the "best" linear approximation to the function at the point.

This lesson will help you explore the linearization function to see why it is a good approximation to the function. First you will find the linearization of a function and graph the function and its linearization on the same set of axes using PLOTR. Then you will evaluate the function and its linearization for a number of values close to the point of tangency, using the SOLVR.

EXAMPLE 1: Consider the function

$f(x) = \sqrt{x}$ at the point x = 4.

Find the linearization L(x) for this function.

Draw the graphs of f and L together on the same set of coordinate axes and make a table of values of both functions for comparison. Use values which approach 4 from the left and the right.

$f(x) = \sqrt{x}$ ==> $f'(x)$ = ________

so $f(4)$ = _____

$f'(4)$ = _____

$L(x)$ = ______________________

Graph f(x) using the default screen and plot L(x) over this graph. Then use SOLVR with $\sqrt{x} = .25x + 1$

x	3.5	3.9	3.99	3.999	4.5	4.1	4.01	4.001
f(x)								
L(x)								

EXAMPLE 2: Now try the same thing for f(x) = sin(x) at x = pi/4.

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EXPLORING THE MEAN VALUE THEOREM ON THE HP-48S

This worksheet is a preview of the Mean Value Theorem which says that under certain condition if you take two points of a function and draw the secant through them there is a point on the graph of the function between the two original points whose tangent is parallel to the secant.

1) Consider the function, $f(x) = .4x^3 + 1.4x^2 + .9x + .7$.
   a) Sketch this graph in the SVR on the grid below. Two grids are provided if you wish to use one a scratch..

   b) What is f(-3.3)? ________________

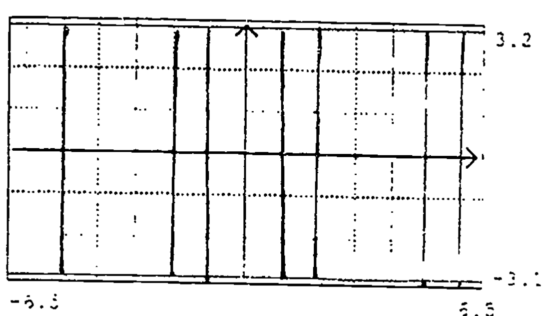
   c) What is f(.9)?________________

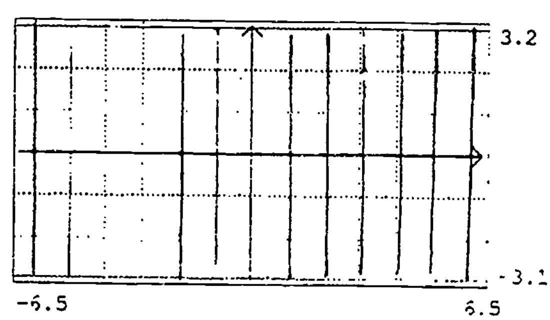
   d) Calculate the slope of the line through those two points.________________

   e) Graph the straight line through those two points on your previous graph.

2) Use your graph draw a straight line which is tangent to your function and parallel to the line through (3.3, f(-3.3)) and (.9, f(.9)).

3) Give an equation of the line you estimate in section 2.





EXPLORING EXTREMA AND INFLECTION POINTS WITH THE HP-48S

In the following calculations, use $g(x) = x^5/10-2*x^4/3+2*x^3/3+2*x^2-2*x+1 = \frac{x^5}{10} - \frac{2x^4}{3} + \frac{2x^3}{3} + 2x^2 - 2x + 1$.

We have observed that when $f'(x) > 0$, $f(x)$ is increasing and when $f'(x) < 0$, $f(x)$ is decreasing. That is, the sign of the derivative gives us information concerning the function. We are only looking at polynomial functions so the derivative can only change signs when it is zero. At a change in sign of the derivative, we get an extrema of the polynomial function.

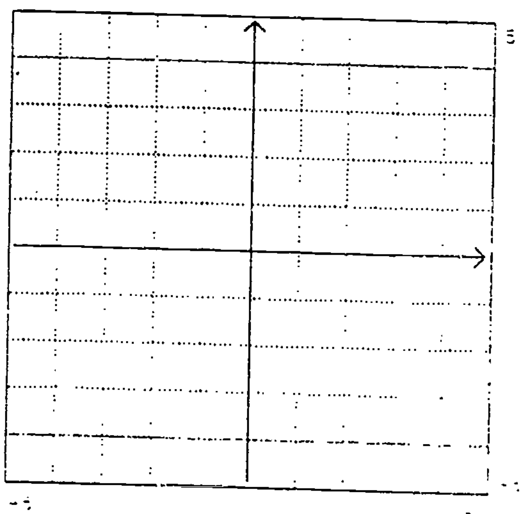
1) In this worksheet, we want to examine the relationship of $f''(x)$ to $f(x)$. We know that if $f''(x) > 0$, then $f'(x)$ is increasing. Let us see what this looks like graphically. On GRID #1 below sketch a graph that goes through the following points with approximations to the indicated slopes which are increasing.

POINT	SLOPE
(-4,4)	-4
(-3,1)	-3
(-2,-1)	-1
(0,-2)	0
(2,-1)	1
(3,1)	3
(4,4)	4

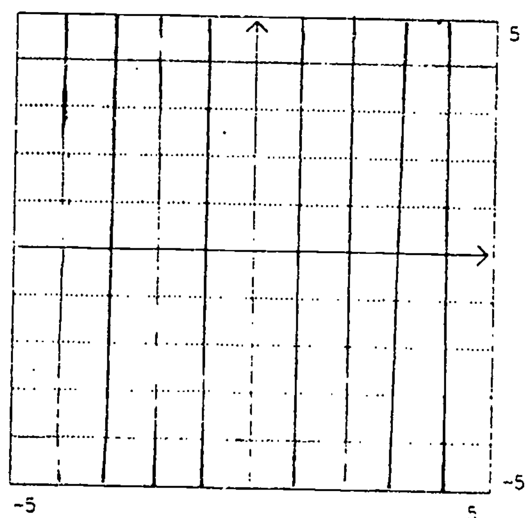
A graph with this shape is called concave up. It occurs when the first derivative is increasing. That is, when the second derivative is positive. Similarly, when the second derivative is negative, that is the first derivative is decreasing, the curve is concave down.

2) On Grid # 2 sketch a graph of $h(x) = 5 - x^2$. Calculate $h''(x)$? ____________________

What is the sign of $h''(x)$? ____________________



GRID #1



GRID #2

OVER

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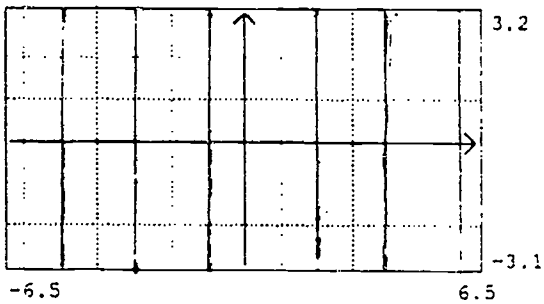
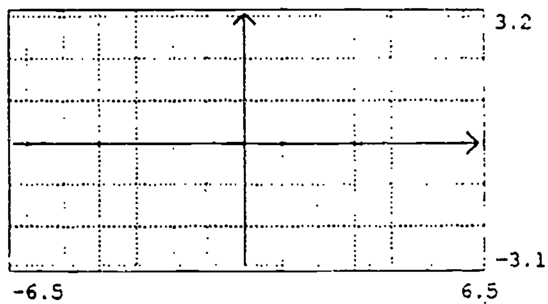
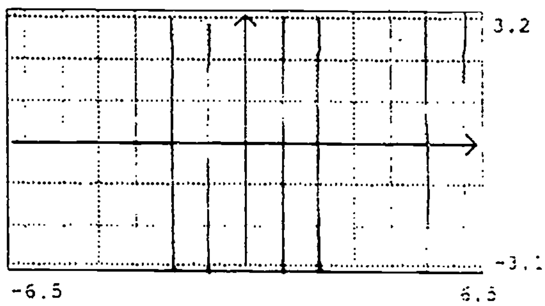
3) In the following, we will examine the graph of the function g(x) above. Use the SVR to graph g(x), g'(x), and g"(x) on the grids below. Use the following pattern. When g(x) is concave up, use dashes; when g(x) is concave down, use a solid line. When g'(x) is increasing, use dashes; when g'(x) is decreasing use a solid line. When g"(x) is positive, use dashes; when g"(x) is negative use a solid line. Draw vertical lines beginning at the zeros of g"(x) through the extrema of g'(x) and on up through g(x). These vertical lines should hit g(x) at points where the concavity changes. These are called inflection points. (I have included two sets of graphs so that you may use one as scrap. Your final answer should be on the right hand set.)

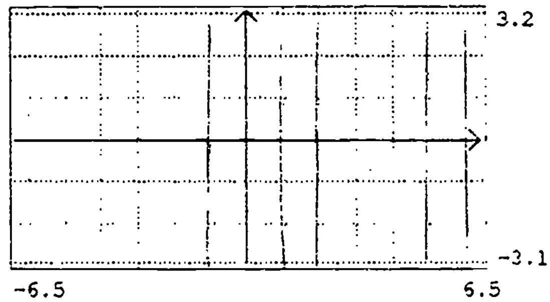
g(x)



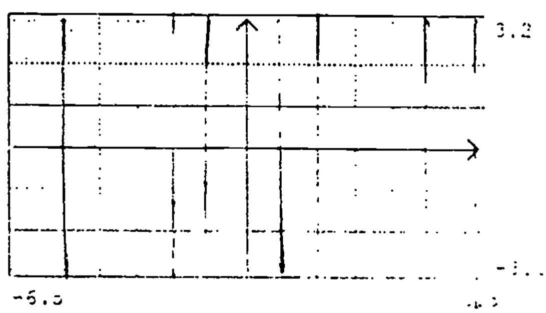


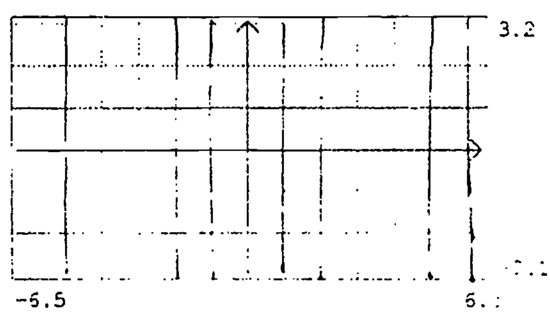
g'(x)





g"(x)





4) Discuss relationships between a function and its second derivative.

NEWTON'S METHOD ON THE HP48

Recall that to solve f(x) = 0 using Newton's method, we take an initial "guess" at the solution, x_0, and then compute succeeding approximations by using

$$x_{n+1} = x_n - \underline{f(x_n)} \;/\; f'(x_n)$$

STEPS TO FOLLOW:

1. Purge X by ===> αX Orange PURGE

2. Enter 'αX

3. Enter the function f(x) (example $x^3 - 4x + 1$)
   ' X^3-4*X+1 ENTER
   ENTER again to duplicate the function

4. αX BLUE ∂ to differentiate one copy of the function (derivative with respect to X).

5. Divide to get f(X) / f'(X)

6. Subtract (from X) to get
   X - f(X) /f'(X)

NOW go to the SOLVE menu to approximate.

7. ORANGE SOLVE
   STEQ (white key) to make this function the current one.
   SOLVR (white key)
   now start evaluating the expression for various X values:
   ENTER X_0, the initial approximation;
   Repeatedly
   ENTER to get another value for X on the stack
   EXPR = ? to get the value of the next approximation;
   Until two consecutive values are "close enough".

Try (a) $x^3 - 4x + 1 = 0$, using $x_0 = 0$; then try 1;

(b) $\sqrt[5]{33}$ by solving $x^5 = 33$ (WHY?)

EXPLORING ROOTS, BOUNDS, AND EXTREME OF A FUNCTION USING THE HP-48S

The function F(X) = 1.5* EXP(-X/2) * SIN(2*X) for x > 0 represents damped harmonic motion.

1. Graph the function in a viewing window with X between -.1 and 6.4, and Y between -1.6 and 1.6.

2. What two curves will bound F(X) above and below?

3. What are the roots of F(X)? Are these the same as the roots of SIN(2*X)? WHY?

4. Do the extrema of F(X) occur at the extrema of SIN(2*X)?

PROGRAMS

This section contains original programs suitable for the precalculus and calculus levels. Programs for both the TI-81 and the HP-48S calculators are included. These worksheets may be duplicated for non-commercial use .

PROGRAM FOR EVALUATING A FUNCTION
(TI-81 Graphics Calculator)

To enter a program into your TI-81 calculator, you must enter the programming mode by striking the key [PRGM] located in the third row, third column of your calculator. To write a program you must cursor to the right [▷] (dark blue key, upper right hand corner of calculator) until you reach the selection EDIT. You will notice that EDIT now becomes darkened. Press the [ENTER] key located at the bottom right corner of your calculator. The following will appear on your calculator screen:

Prgm1:

:

You are now ready to type in the *name* of your program on the line Prgm1: immediately after the colon.

Prgm1: [2nd] [ALPHA] [SIN]^{E} [6]^{V} [MATH]^{A} [)]^{L} [COS]^{F} [X|T] [ENTER]

You are now ready to enter your first program.

: [PRGM] [▷|I/O] [ENTER| 1:DISP] [ALPHA] [+]$^{"}$ [X|T] [ALPHA] [+]$^{"}$ [ENTER]

: [PRGM] [▷|I/O] [▽| 2:INPUT] [ENTER] [X|T] [ENTER]

: [2nd] [VARS]$^{\text{Y-vars}}$ [ENTER] [STO▷] [X|T] [ENTER]

: [PRGM] [▷|I/O] [ENTER| 1:DISP] [ALPHA] [+]$^{"}$ [2nd] [VARS]$^{\text{Y-vars}}$ [ENTER]

[2nd] [MATH]$^{\text{TEST}}$ [ENTER| 1:=] [ALPHA] [+]$^{"}$ [ENTER]

: [PRGM] [▷|I/O] [ENTER| 1:DISP] [X|T] [ENTER]

This is the last line of the program. Your program should look like:

```
Prgm1:EVALFX
:Disp "X"
:Input X
:Y1→ X
:Disp "Y1="
:Disp X
```

To store the program and leave the programming mode, press the following keys:

QUIT
[2nd] [CLEAR]

To run the program you must:

1. Enter the function you want to evaluate by first pressing [Y=] located in the upper left corner of the calculator to bring up the function display and then typing in the rule of the function you wish to evaluate.
2. Press [PRGM] (Note: EXEC is darkened)
3. Press [ENTER] (Prgm1 appears in the screen of the calculator)
4. Press [ENTER] again. The following will appear on the screen:

X
?

5. Enter the value for X for which you wish to evaluate the function and press [ENTER].
6. The numerical quantity displayed on the right side of the screen is the value of the function for the X you entered in step 5.

SYNTHETIC DIVISION (TI-81)

SYNDIV divides a polynomial P(x) of degree n by (x - c) after entering n, c, and the coefficients of the terms of P(x) where P(x) is in descending order.

NOTE: Columns two and four are directions for locating symbols.

Program SYNDIVL   PRGM EDIT 3 SYNDIV

:Disp "N"		:I - 1→J	
:Input N		:Disp J	
:Disp "C"		:Input B	
:Input C		:AC+B→A	
:1→I		:DS<(I,1)	PRGM CTL
:Lbl 1		:Goto 2	
:0→{x}(I)	{x} on 0 key	:Disp "REM"	
:IS>(I,N)		:Disp A	
:Goto 1		:Pause	PRGM CTL
:N→I		:Disp "QUOT"	
:Disp "COEF"		:N→I	
:Disp N		:Lbl 3	
:Input A		:Disp {x}(I)	
:Lbl 2		:DS<(I,1)	
:A→{x}(I)		Goto 3	
:Disp "COEF"			

THE QUADRATIC FORMULA (TI-81)

Program QUADF	PRGM EDIT QUADF
:ClrHome	PRGM I/O
:Disp "A"	PRGM I/O
:Input A	PRGM I/O
:If A=0	PRGM CTL ; 2nd TEST
:Goto 4	PRGM CTL
:Disp "B"	
:Input B	
:Disp "C"	
:Input C	
:$B^2-4AC\to D$	
:If D≤0	PRGM CTL ; 2nd TEST
:Goto 1	PRGM CTL
:$(-B+\sqrt{}D)/2A\to R$	
:$(-B-\sqrt{}D)/2A\to S$	
:ClrHome	
:Disp "THE ROOTS ARE"	
:Disp R	
:Disp "AND"	
:Disp S	
:Goto 3	PRGM CTL
:Lbl 1	PRGM CTL
:If D<0	PRGM CTL ; 2nd TEST
:Goto 2	
:$(-B)/2A\to R$	
:ClrHome	
:Disp "THE REPEATED"	
:Disp "ROOT IS"	
:Disp R	
:Goto 3	
:Lbl 2	
:$-B/2A\to R$	
:$(\sqrt{}-D)/2A\to I$	
:ClrHome	
:Disp "COMPLEX ROOTS"	
:Disp "ONE IS"	
:Disp "R="	
:Disp R	
:Disp "IM="	
:Disp I	
:Disp "ENTER FOR NEXT"	
:Pause	PRGM CTL
:ClrHome	
:Disp "SECOND IS "	
:Disp "R="	
:Disp R	
:Disp "IM="	
:$-I\to I$	
:Disp I	
:Lbl 3	
:End	PRGM CTL
:Lbl 4	
:Disp "NOT QUADRATIC"	

QUADF solves quadratic equations of the form

$$Ax^2 + Bx + C = 0$$

after entering A,B and C. Real and complex solutions are reported.

NOTE: Columns two and four are directions for locating symbols.

AUTOSCALE GRAPHING (TI-81)

With a function stored in y_1 and values entered for x_{min} and x_{max}, AUTOSCL automatically chooses appropriate values for y_{min} and y_{max}, sets the graphing window, and graphs y_1.

NOTE: Columns two and four are directions for locating symbols .

Program AUTOSCL   PRGM EDIT AUTOSCL

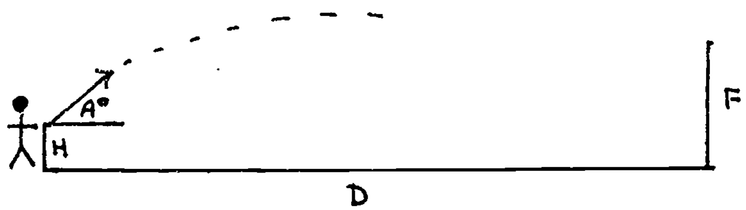
:All-Off	2nd Y-VARS OFF	:Lbl 1	PRGM CTL
:Disp "A"	PRGM I/O	$:X+H \rightarrow X$	
:Input A	PRGM I/O	$:IF\ Y_1 \leq C$	2nd TEST
:Disp "B"		$:Y_1 \rightarrow C$	
:Input B		$:If\ Y_1 \geq D$	
$:A \rightarrow X_{min}$	VARS RNG	$:Y_1 \rightarrow D$	
$:B \rightarrow X_{max}$	VARS RNG	:IS>(I,23)	PRGM CTL
$:(B-A)/5 \rightarrow X_{scl}$	VARS RNG	:Goto 1	PRGM CTL
$:(B-A)/23 \rightarrow H$		$:C \rightarrow Y_{min}$	
$:A \rightarrow X$		$:D \rightarrow Y_{max}$	
$:Y_1 \rightarrow C$	2nd Y-VARS	$:(D-C)/5 \rightarrow Y_{scl}$	
$:C \rightarrow D$		$:Y_1$-On	2nd Y-VARS RNG
$:1 \rightarrow I$		:DispGraph	PRGM I/O

BASEBALL : OVER THE "BIG GREEN WALL" ? (TI-81)

This program for the TI-81 will graphically show whether a ball hit with an initial velocity of V ft/sec at an angle of A degrees will go over a fence F feet high D feet away. You will be asked to input:

- A the angle in degrees
- V the initial velocity (ft/sec)
- H the bat height
- F the fence height (ft)
- D the distance to the fence from the batter (ft)

```
Prgm: BALLGAME
: Disp "ANGLE"
: Input A
: Disp "VELOCITY"
: Input V
: Disp "BATTER"
: Input H
: Disp "FENCES"
: Input F
: Disp "DISTANCE"
: Input D
: Param
: Deg
: Connected
: 0→Tmin
: 5→Tmax
: .5→Tstep
: -.5→Xmin
: D+20→Xmax
: 40→Xscl
: -.5→Ymin
: 100→Ymax
: 20→Yscl
: "D"→X1T
: "FT(T≤1)"→Y1T
: "VTcosA"→X2T
: "H+VTsinA-16T²"→Y2T
: DispGraph
: End
```



Recall that

80mph ≈ 117.3ft/sec
90mph ≈ 132.0ft/sec
100mph ≈ 146.7ft/sec
110mph ≈ 161.3ft/sec

THE BINOMIAL SERIES (TI-81)

The following program will set up "Y=" and the range to show (x+1)^K and the 2nd, 4th, and 6th degree polynomial approximations for (x+1)^K.

```
Prgm6:BINOM
:Disp"Y1 =(X+1)^K"
:Disp"Y2 =2DEG POLY"
:Disp"Y3 =4DEG POLY"
:Disp"Y4 =6DEG POLY"
:Disp"K="
:Input K
:"(1+X)^K"→Y1
:"1+K*X+(K(K-1)/2)X^2"→Y2
:"Y2+(K(K-1)(K-2)/6)X^3+(K(K-1)(K-2)(K-3)/24)X^4"→Y3
:"Y3 +(K(K-1)(K-2)(K-3)(K-4)/120)X^5
      +(K(K-1)(K-2)(K-3)(K-4)(K-5)/720)X^6"→Y4
:-2→Xmin
:2→Xmax
:.5→Xscl
:-5→Ymin
:5→Ymax
:1→Yscl
```

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GRAPHING THE DERIVATIVE OF A FUNCTION (TI-81)

OBJECTIVE----The objective of this exercise is to enter a program into the calculator that will graph the derivative of a function.

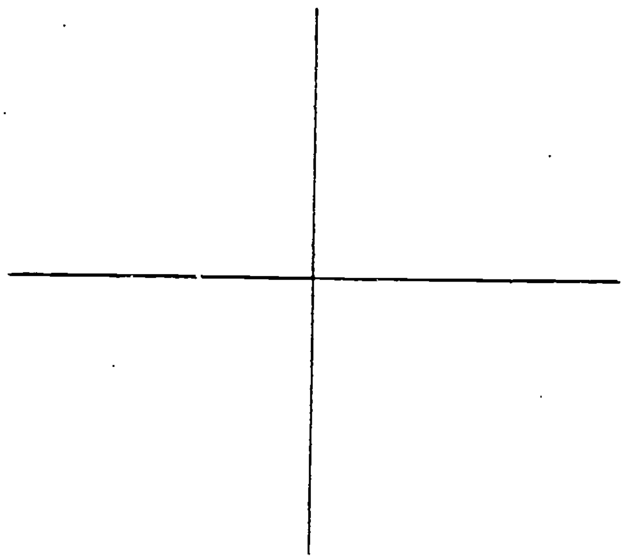
Enter the program at the right into your calculator.

CLASS EXAMPLES

For the following functions a) determine the derivative of f, b) enter f in y_1 and graph f over the given domain and range, c) use the program to obtain a graph of f and f', and d) draw and label both graphs on the grid provided.

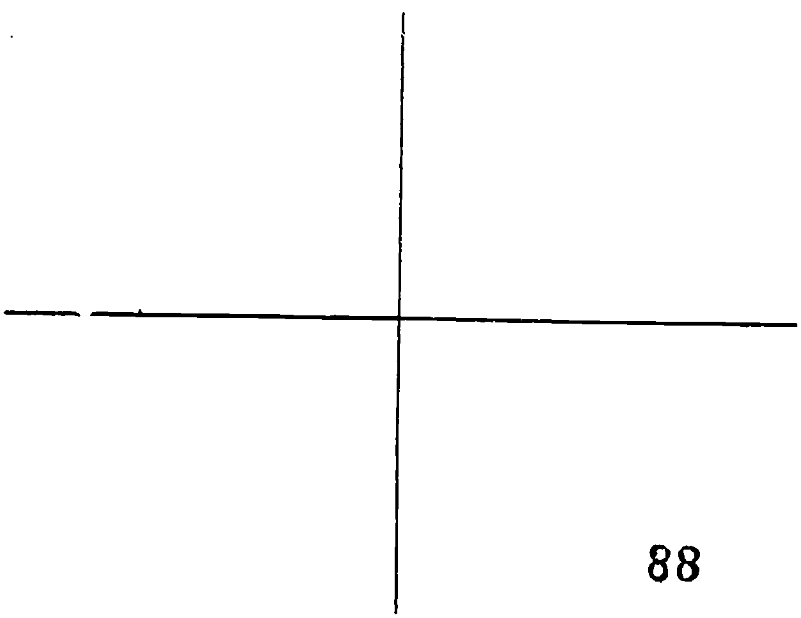
1. $f(x) = x^2 - 3x + 4$; standard values

   $f'(x) =$



2. $f(x) = 2\cos x$; $-6.28 \le x \le 6.28$ and $-3 \le y \le 3$

   $f'(x) =$



```
Prgm: DERVPLOT
All-Off
Y1-On
(Xmax-Xmin)/100→H
.001→D
DispGraph
Pause
Xmin→X
Lbl 1
NDeriv(Y1,D)→Y
Pt-On(X,Y)
X+H→X
If X≤Xmax
Goto 1
Pause
```

HOMEWORK EXERCISES

1. $f(x) = 2x^2 - x + 3$; standard values

   $f'(x) =$

2. $f(x) = x^3 - 2x$; $-3 \leq x \leq 3$ and $-5 \leq y \leq 5$

   $f'(x) =$

3. $f(x) = 1/x^2$; $-5 \leq x \leq 5$ and $-10 \leq y \leq 10$

   $f'(x) =$

4. $f(x) = 2 + \sin x$; $-6.28 \leq x \leq 6.28$ and $-2 \leq y \leq 4$

   $f'(x) =$

Solving Differential Equations by Euler's Method on the TI-81

The following program uses Euler's Method (Increment Method) for solving first order differential equations. Assuming the D. E. y' = f(x,y), store f(x,y) in Y1. The program allows you to select initial x and y values and to choose dx. It will give you the new x and y value at each step then ask if you want to continue for the next x and y.

```
Prgm 2: Euler
:Disp "THIS PROG ASSUMES DY/DX IN Y1"
:Disp "INPUT X"
:Input X
:Disp "INPUT Y"
:Input Y
:Disp "INPUT DX"
:Input D
:Lbl B
:Y + Y1*D → Y
:X + D → X
:Disp "X="
:Disp X
:Disp "Y="
:Disp Y
:Disp "IF DONE 1, IF NOT DONE 0"
:Input T
:If T = 1
:Goto A
:Goto B
:Lbl A
:END
```

Example
y' = xy - y^2   Y(0) = 1   Find y when x = 1 use dx = .2

press Y=
enter on Y1 = x(alpha)y - (alpha)y^2
press 2nd CLEAR
press PRGM
(assuming the program has been stored under Prog 2) press 2
ENTER
You will be asked to in put x, y, and dx.
After you input these values, you will be given that
x = .2 and y =.8
Since we are not finished enter 0
You will be given
x = .4 and y = .704
Continue this process until you get x=1 and y = .67229
Enter 1 to end the process.

Solving Differential Equations by the Runge-Kutte Method on the Texas Instrument TI-81

The following program uses the Runge-Kutte Method for solving first order differential equations. Assuming the y' = f(x,y), store f(x,y) in Y1. The program allows you to select initial x and y values and to choose dx. It will give you the new x and y value at each step then ask if you want to continue for the next x and y.

```
Prgm 3: RK
:Disp "THIS PROG ASSUMES DY/DX IN Y1"
:Disp "INPUT X"
:Input X
:Disp "INPUT Y"
:Input Y
:Disp "INPUT DX"
:Input D
:X→P
:Y→Q
:Lbl B
:Y1→K
:(P+(D/2))→X
:(Q+(D*K)/2)→ Y
:Y1→L
:(Q+(D*L)/2)→Y
:Y1→ M
:(P+D)→ X
(Q+D*M)→ Y
Y1→N
:(K+2L+2M+N)/6→R
:X →P
:(Q+D*R)→ Y
:Y →Q
:Disp "X="
:Disp X
:Disp "Y="
:Disp Y
:Disp "IF DONE 1, IF NOT DONE 0"
:Input T
:If T = 1
:Goto A
:Goto B
:Lbl A
:END
```

Example
y' = xy - y^2   Y(0) = 1 Find y when x = 1 use dx = .2
(See instructions from Euler , but select the RK program)
You will end with x = 1 and y = .75112

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THE TRAPEZOIDAL RULE FOR INTEGRATION (TI-81)

OBJECTIVE----The objective of this exercise is to program the calculator to approximate definite integrals by using the Trapezoidal Rule.

CLASS EXAMPLES

1. Enter the program at the right into your calculator.

2. Put each of the following functions into Y_1 and for each one use the program to evaluate the integrals.

   a) $\int_1^2 \frac{1}{x}\,dx$

   b) $\int_1^4 \sqrt{1+x^3}\,dx$

   c) $\int_0^{\pi} \sin\sqrt{x}\,dx$

```
Prgm2:TRAPRULE
:ClrDraw
:Disp "LEFT ENDPOINT"
:Input A
:Disp "RIGHT ENDPOINT"
:Input B
:0→S
:50→N
:(B-A)/N→W
:1→I
:A→X
:Lbl 1
:Y1→Z
:A+IW→X
:.5(Z+Y1)W+S→S
:IS>(I,N)
:Goto 1
:Disp " "
:Disp "TRAP APPROX"
:Disp S
```

HP48S calculator program for gradient vector:

Type:

```
<<F space Z space ∂>>  ENTER
'FZ STO
<<F space Y space ∂>>  ENTER
'FY STO
<<F space X space ∂>>  ENTER
'FX STO
<<FX space FY space FZ>>  ENTER
'GRADIENT  STO
```

Example: To obtain the gradient of the function $f(x, y, z) = x^2 + y^2 + z^2$, type

```
'X^2 + Y^2 + Z^2   ENTER
'F  STO
```

Now press GRADIENT to obtain 2x, 2y, 2z.

*************************************** * * ******

To do a double integral $\int_{x=a}^{x=b}\left[\int_{y=c}^{y=d} f(x,y)\,dy\right]dx$, we need the folowing program:

****

Type: << A space B space F space X space $\int$ >> ENTER

'INFAB STO

<< C space D space G space Y space $\int$ >> ENTER

'INGCD STO

1 ENTER	'G STO	1 ENTER	'F STO
2 ENTER	'D STO	3 ENTER	'C STO
4 ENTER	'B STO	5 ENTER	'A STO

****

To integrate f(x,y) = cos(x + y), x from 0 to 1, and y from 0 to x^2, we have the double integral $\int_0^1\left[\int_0^{x^2}\cos(x+y)\,dy\right]dx$. This is done on the calculator as follows:

Type: ' COS (X + Y)' ENTER

'G STO

(The function is stored as G since we are integrating over y first, and G is the function that is integrated over y.)

0 ENTER 'C STO 'X^2' ENTER 'D STO

0 ENTER 'A STO 1 ENTER 'B STO

(You have now entered all the upper and lower limits. Note that C and D are associated with the integral over y, and A and B are associated with the integral over x.)

Press INGCD EVAL 'F STO

(This does the integral of G from y = C to y = D, and it stores the result under the label F.)

Press INFAB EVAL

(This does the integral of F from x = A to x = B.)

Noting that the calculator does not produce a number for you (because the integrand F is too complicated to do analytically), type:

blue arrow EVAL

(This causes the calculation to be done numerically.)

Your screen will show the value .15

****

To check whether we could have done the x integration first, we need to evaluate:

$\int_0^1\left[\int_{x=\sqrt{y}}^{x=1}\cos(x+y)\,dx\right]dy$. On your calculator type:

' COS (X + Y) ENTER 'F STO

'$\sqrt{Y}$ ENTER 'A STO 1 ENTER 'B STO

0 ENTER 'C STO 1 ENTER 'D STO

INFAB EVAL 'G STO

INGCD EVAL blue arrow NUM

The value .15 will again show on your screen, which checks with the previous calculation.

GETTING ACQUAINTED

This section contains several hand-outs that can be used to help students (or faculty !) get acquainted with the TI-81, HP-48S or the HP-48G. Enough detail is provided to introduce the reader to the important features of each machine. The topics are not exhaustive. Additional features await futher discovery.

TI-81 Graphics Calculators: Selected Topics to Enhance Precalculus and Calculus Instruction

1) Features:
   - menu-driven
   - can save and graph up to 4 functions
   - can trace over any function
   - built-in piecewise, parametric graphing
   - can handle up to three 6x6 matrices
   - thirty-seven program slots

2) Recognizes order of operations, e.g 2/3*4 [enter] gives 2.666...7

However, also recognizes monomials
   e.g 4 [sto] x then press [enter]
        2/3x [enter] gives as answer .1666...7

3) Recursion
   Let's make a table for the powers of two:
      2*2 [enter]; now press *2 again to see on screen
      Ans * 2 ; keep pressing [enter] to generate the table

Examples: which is better -- $1000 at 6% compounded annually or quarterly?
   compare for three years.

   1000* 1.06 [enter]
   then *1.06 and press [enter] twice

Now 1000*(1+ .06/4) [enter] then *(1 + .06/4) and press [enter] 11 more times.

Question: if my credit card has an APR of 21%, how long before my initial debt doubles?

4) Generating a Random number

on [Math] Menu, press ▶ to get to PRB section; press 1 or [enter] to select RAND; keep pressing [enter] to generate as many random numbers between 0 and 1 as you want.

6 RAND or RAND 6 gives a number between 0 and 6 (but not equal to 6)

in general, RAND M, where M is any positive whole number, will generate a random real number x, where 0 < x < M

use IPART (on the MATH Menu) to get just the integer part, so IPART RAND 6 [enter] gives 0,1,2,3,4,or 5

IPART RAND 6 + 1 gives 1,2,3,4,5,or 6

can use this for 1) simulations 2) randomly collecting homework

5) Defining Functions

press [Y=] to see slots for four functions

type in $5x^3 - 4x^2 + 3x - 2$ for Y_1 ; press [enter] [2nd] [quit]

recall before we stored 4 in x; press x to see that; now press [2nd] [vars] select Y_1 by pressing [enter] (pressing [enter] will select whatever is highlighted on any menu) ; press [enter] again to see value of the function for x=4 (19502)

now let's store a new value for x:
1 [sto] x [enter];
then [2nd] [vars] [enter] [enter] again gives 2

while this process requires several keystrokes, it's still better than typing in all the operations on a regular calculator!

Now press [Y=] again. use ▼ to get to Y_2 and type in $Y_2 = x^2 - 5x + 6$

Now [2nd] [VARS] press 2 [enter] to get the value of Y_2 for x=1 which is 2

6) Computing a limit numerically

Press [Y=] and clear out Y_1 and Y_2

let $Y_1 = (x+2)/(x^2-4)$; [2nd] [quit]. Using the procedure outlined above, compute the values of Y_1 for x = 1.8, 1.9, 1.99, 2.3, 2.1, 2.01

7) MODES settings: always leave in radians; if you want to do a computation in degrees, you can find degree sign on the [MATH] menu. For example, to compute the sine of 30 degrees, press the [sin] key, type in 30, then press [MATH] highlight the degree symbol using the down arrow ▼ and press [enter]. Press [enter] again to complete the computation.

8) Graphing

Just press [GRAPH] to see a graph of the function stored as Y_1

Not so good, so press [ZOOM] and select 2 (ZOOM IN)

still not so good. Lets press [RANGE] to see the values.

The TI 81 has 96 pixels across and 64 down its screen. So starting at XMIN, each pixel is numbered across by increments of Δx, where $\Delta x = \frac{XMAX - XMIN}{95}$. Similarly, $\Delta y = \frac{YMAX - YMIN}{63}$

If the increments are nice decimals, like .1 , then you see a nice graph.

(Why does the asymptote appear? Well, in this case x 2 is not in the domain of the function. The calculator knows this; however, based on the way the x-values are currently assigned to the pixels horizontally, no pixel may represent 2 at all. So, there is a pixel for an x a little smaller than 2 and a pixel for an x a little larger than 2, and the calculator dutifully connects the points it determines for those two x values. It never has to do a computation for 2 itself.)

So let's change to range settings to XMIN = -4.7 ; XMAX = 4.8 ; YMIN = -3.1; YMAX = 3.2

To do this, when we press [RANGE], the cursor should be at XMIN -- just type in -4.7 (Remember the [(-)] key is not the same as the subtraction key; mixing them up will really mess up a graph or a computation!). After you have typed in -4.7, just press [enter]; then type in 4.8 for XMAX, press [enter]. You can either press [enter] or the ▼ to go over XSCL. Repeat this procedure for the y values.

Now press [2nd] [QUIT] and [GRAPH]. press [TRACE] and use left arrow ▶ to trace over curve to the left side. Notice the screen scrolls with the cursor. You can see that all the x-values differ by .1 now. As you trace over the curve, you'll see the x and y values at the bottom of the screen; note when x=2, the y is blank.

9) Solving Equations graphically

[Y=] and [CLEAR] out Y_1

let $Y_1 = x^3 - 2x$ (the cube sign is on the MATH menu)

$Y_2 = 2 \cos x$

Graph one at a time, so put the cursor on the = sign for Y_2 and press [enter]
then [GRAPH]. Placing the cursor on the equal sign for a function and pressing [enter] will turn it on or off -- you can tell by the box that appears around the equal sign when the function is turned on.

If all of the stored functions are on, they will all be graphed in order sequentially (or simultaneously if this option is highlighted on the [MODES] menu). So, now turn on the other function and [GRAPH] again to see them together.

Press [TRACE] ; the left and right arrows move along a curve; up and down arrows switch from one curve to the other.

The curves apparently intersect in two places; let's find the right answer first. Press [ZOOM] and choose BOX. Use the four arrows to position the crosshair for the upper left corner of the box and press [ENTER] then move the crosshair to whatever you would like for the lower right corner. Press [ENTER] again to re-do the graph. Use [TRACE] to compare the X and Y values for the point of intersection. This zooming procedure may be done several times to increase the accuracy of the answer.

Now try to find the left answer -------? Press [ZOOM] 6 to return to the standard viewing window. Now repeat the above procedure around the left intersection. What do you finally see??? !

ANOTHER EXAMPLE:
Press [Y=] and clear out Y_1 AND Y_2 by highlighting each in turn and pressing [CLEAR]. Now replace them with Y_1 = ln x + x and Y_2 = 1. We're trying to solve ln x + x = 1, using the above procedure.

10) Comparing Graphs:

Since we can graph up to four functions at a time we can easily show horizontal and vertical shifts, e.g. y = |x|, y= |x| +1, y = |x| -2, y = |x-1|, y = |x-2|

11) Factoring:

Graph $y= x^2 - 5x + 6$ and note that the two x-intercepts correspond to the two factors (x-2) and (x-3).

For any polynomial f(x), f(a) = 0 if and only if (x-a) is a factor.

We will also use the fact that every odd degree polynomial has to have at least one real root. Factor $y = x^3 - 3x + 4$.

Graph it and we see one negative root. [ZOOM] BOX as before to get the root to about 3 accurate decimal places. Leave the TRACE cursor on your best approximation for that root. Now press [2nd] [QUIT]. If you press [X] [ENTER] again, you will see the value of the root on your screen. now press [STO] A to store the root as A.
Press [Y=] and type in Y_2 = Y_1 / (x-A). (Remember you can get Y_1 from the YVARS menu). Press [ZOOM] 6 to see both the original cubic and the new parabola graphed together. That parabola represents the second factor. Turn off Y_1 and regraph Y_2. ZOOM in to get the coordinates of the y-intercept and the vertex. For example, after zooming in on the vertex, press [TRACE] and position the TRACE cursor as close as possible to the vertex. When you press [2nd] [QUIT] to return to the main screen, press [X] [ENTER] to see the x coordinate; then press [STO] [H]. Now press [2nd] [YVARS] 2 [ENTER] to select Y_2 then press [ENTER] again to compute the actual y-value for your x (you haven't moved the TRACE cursor, so that x is still remembered.) Now press [STO] K. The intercept is earier to find (HOW?). Store the intercept as D. Use the parabola form
$y = C(x-H)^2 + K$; if you substitute (0,D) for (x,y) in that formula, you'll find $C = (D-K)/H^2$. Store the value C as well.

Finally we can graph $Y_3 = C(x-H)^2 + K$ and $Y_4 = (x-A)*(Y_3)$ to compare our results with the original. You can turn various y's on and off to see how well they agree.

12)RATIONAL FUNCTIONS

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First, an infinite limit! compare the graphs of $y_1 = x^3 - 4x^2 + 5$ and $y_2 = x^3$.

Graph both on the standard screen; then change the RANGE so that x and y are both from -50 to 50. How about -100 to 100? What does that mean for

$$\lim_{x \to \infty} \frac{x^3}{x^3 - 4x^2 + 5}$$

How about comparing with $y_3 = x^3 - 3x^2 + x + 3$?

Now we'll graph $y_1 = \dfrac{x^2 - 1}{x^2 - 2x - 8}$

also $y_1 = 2x + 1 + \dfrac{1}{(x-1)(x+2)}$

and $y_1 = \sin x + \dfrac{1}{x+1}$

In each case, we're looking at the vertical asymptotes and general trends in the graphs. We may want to find "friendly windows" to eliminate the asymptotes, based on multiples of 95 as the difference between XMIN and XMAX, and multiples of 63 as the difference between YMIN and YMAX.

13) Look at $y_1 = x^3 - 2x^2 + x - 30$.

all cubic curves look like

What is a complete graph of the above?? Try zooming in at the right place. Calculus fans may have some ideas.

14) Investigating a function and its Derivative

graph $Y_1 = x^2 - 5x + 6$

$Y_2 = 2x - 5$

$Y_3 = Y_1 \, (Y_2 < 0)$ < is on the $\boxed{\text{TEST}}$ menu

How about the function $Y_1 = x^3 - x^2 + 2x + 1$ with $Y_2 = 3x^2 - 2x + 2$?

The TI-81 also computes a numerical derivative. For Y_2, you could use instead NDeriv(Y_1 , , .1) (NDeriv is found on the $\boxed{\text{Math}}$ Menu; the .1 is a computation tolerance; a larger number gives a worse approximation.)

15) We can SOLVE inequalities using the above idea. How about $|x+1| > 2$?

16) one last rectangular example:

Let Y_1 = sin x
Y_2 = NDERIV(sin x, .1) the Numerical derivative is found on the MATH menu #8.

Now graph Y_3 = cos x

17) PARAMETRIC EQUATIONS change $\boxed{\text{MODES}}$ from FUNCTION to PARAMETRIC

The ferris wheel problem is in the TI manual. Basically, a person on the ground wants to throw something to someone on a ferris wheel. The parametric equations below are used to describe the motion of the wheel and of the object. (Note in this case, leaving out some of the operation symbols may mess up the computations; 'tis better to be safe than sorry, so put them all in!)

X_{1T} = 20 cos(π *T/6)
Y_{1T} = 20 sin(π *T/6) +20

X_{2T} = 75 - 30 T
Y_{2T} = -16 T^2 + 30 * $\sqrt{3}$ * T

On the RANGE, T has the values 0 to 12; x -25 to 100, y -1 to 45. (Note, in PARAMETRIC mode , this T will now appear on the RANGE menu. It didn't when we used Function mode.)

Graph sequentially, then change to SIMULTANEOUSLY to see the true motion according to time. TRACE will also show all three values, T, x, and y.

18) POLAR EQUATIONS

have to be typed in parametrically as x_T = r cos t and y_T = r sin t

Investigate rose curves r = sin nT or r = cos nT for n even integers, odd integers. How about n as a decimal -- something you could never do before at a blackboard! Be sure that T runs from 0 to 6.28...

19) graphing logs

obviously y = ln x or y = log x

but at last a use for the base change formula $\log_b a$ = ln a/ ln b to graph $Y_1 = \log_2 x$ use instead Y_1 ln x / ln 2

20) SPLIT DOMAIN FUNCTIONS

$$f(x) = \begin{cases} x+2, & x<3 \\ x & x \geq 3 \end{cases}$$

type in as Y = (x+2)(x<3) + (x)(x≥ 3)

DOT mode is best.

21) to shade in the area between two curves

First clear out all the $\boxed{Y=}$ functions.

find the SHADE command on the $\boxed{DRAW}$ menu. First select CLRDRAW $\boxed{enter}$ to clear out previous graphs. Then return to the DRAW menu and select SHADE

the correct syntax is SHADE(lower function, upper function)

let's do SHADE(sin x, cos x) then press $\boxed{enter}$

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To enter a matrix into your calculator. (TI-81)

MATRX	
→	Select edit mode.
Enter	Select 1, 2, or 3; matrix A, B, or C.
2	These 3 steps set the matrix to be 2X3.
→	
3	
↓	
4 ↓	These six steps set up the matrix, $\begin{pmatrix} 4 & 2 & 3 \\ 1 & 1 & 5 \end{pmatrix}$.
2 ↓	
3 ↓	
1 ↓	
1 ↓	
5 ↓	
2^{nd}QUIT	

To display the matrix.

2^{nd}[A]

ENTER

To multiply two matrices.

2^{nd}[A]	The result is the product, [A][B].
2^{nd}[B]	
ENTER	

The inverse of a matrix.

2^{nd}[A]	The calculator displays $[A]^{-1}$. Enter will display the inverse of the matrix. To multiply by a second matrix, enter it's symbol next to $[A]^{-1}$.
x^{-1}	

HEWLITT PACKARD 48S GRAPHING CALCULATOR

I. Press ON (lower left hand corner) to turn the machine on. (This is also the "panic" button in case something goes wrong.)

When typing, you can use the left arrow button to delete typing mistakes. (An alternate thing to do if the "ON" button does not work as a panic button is to delete your input line.) The left and right triangular pointers move the cursor.

To have numbers showing to two decimal places, press

orange arrow button   orange Modes
2 FIX (This is the small button under the word "fix" in the menu.)

To have angles treated as radians, press

orange arrow button   orange Modes
NXT NXT RAD (the small button under the word "rad" in the menu)

To eliminate the lowest stack level from the screen, press

orange arrow button   orange Drop

To turn the calculator off, press

blue arrow button   ON

II. To use this calculator as a basic calculator, we can use one of two methods:

A. REVERSE POLISH proceedure: Press

6	ENTER	
4	+	The display says 10.
6	ENTER	
4	−	The display says 2.
6	ENTER	
4	÷	The display says 1.50 (To enter −6 instead of 6, press 6 ± ENTER)
2 ±	ENTER	This enters −2.
3	y^x	The display says − 8.

Where only one number is needed for input, we can proceed as follows: Press

1	orange arrow e^x	The display shows 2.72 ($\approx e^1$).
3.1	cos	The display shows the value of cos (3.1).
9	$\sqrt{x}$	The display shows 3.

B. CONVENTIONAL proceedure: Press:

' 6 + 4	EVAL	The display shows 10.
' 6 − 4	EVAL	The display shows 2.
' 6/4	EVAL	The display shows 1.5.
' 2 y^x 3	EVAL	The display shows 8.
' orange arrow e^x 1	EVAL	

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' cos 3.1 EVAL — The display shows 2.72

The display shows the value of cos(3.1).

' $\sqrt{x}$ 9 EVAL — The display shows 3.

III. To plot a graph: Let us plot $y = x^2 - 3x + 1$: (Note that when it says to type an alphabetical letter, it really means to tap the α button and then the button that goes with the alphabet letter.) Press

```
'X    y^x    2 - 3 * X + 1    ENTER
orange arrow button    orange plot    STEQ
```

(STEQ is found in the menu on the screen; press the small button under the word)

PLOTR_{menu} ERASE_{menu} DRAW_{menu}

(Now observe your graph; when you are done, press the ON button.)

IV. To take a derivative, first enter your function. Then press

```
'X    ENTER
blue arrow button    ∂
```

(Observe the result $2x - 3$ on the screen.)

Note: If a value is stored as X on the VAR menu, you will see a numerical answer for the derivative. To eliminate this X, press 'X orange PURGE before taking the derivative.

V. To integrate a function: press

```
' ∫ (1,2,X y^x 2-3*X+1, X)    ENTER

EVAL    EVAL
```

(This evaluates the integral $\int_1^2 (x^2 - 3x + 1)\,dx$. To do more complicated integrals, tap the blue arrow button and then the -> num key instead of EVAL.

If an indefinite integral of a simple function is desired, press:

```
'∫(0,X, T y^x 2-3*T+1, T)   ENTER
EVAL    EVAL
```

(This evaluates the integral $\int_0^x (t^2 - 3t + 1)\,dt$.)

VI. To find where a function crosses the x axis, type in the function as in III. Then press

```
orange arrow button    orange solve
STEQ    SOLVR
```

(both of these are in the menu on the screen)

Take a guess, say $x = 3$. Press

```
3    small key under the x in the menu
orange arrow button    small key under x in menu
```

The screen says that a root is $x = 2.62$

VII. To find f(x) for a specific value of x, do all the steps in part VI up to and including typing in the 3. Then press

```
small key under EXPR= in the menu
```

The display will show the value of the function at $x = 3$.

VIII. If you desire a more complete analysis of a graph, proceed as follows: As an example, we take $f(x) = x^3 -$ x
First plot the graph according to III.

' X y^x 3 − X ENTER
orange arrow orange plot $STEQ_{menu}$ $PLOTR_{menu}$ $ERASE_{menu}$ $DRAW_{menu}$

A. If you wish a closer view of the central part of the graph, tap

$ZOOM_{menu}$ xy_{menu} .2 ENTER

This sets the first tick mark on the x and y axes equal to .2(1) = .2, respectively.

B. If you wish to know the coordinates of a point, use the four triangular buttons to move the + marker on the screen to a desired point (such as x = 0). Press

$COOR_{menu}$

Observe the coordinates printed at the bottom of the screen. Tap any key in the top row; then tap

FCN_{menu} $ROOT_{menu}$

Observe the 0.00 on the screen (one root). Move the marker to the right, near where x = 1. Press

$ROOT_{menu}$

and observe the 1.00 on the screen (another root). Do the same for the third root (x = −1).

C. If you wish to know the slope at a point, put the marker near the curve for that point. Press

$SLOPE_{menu}$

and observe the numerical value of the slope printed at the bottom of the screen.

D. If you wish to know the coordinates of an extremum point, place the marker near the extremum point. Press

$EXTR_{menu}$

and observe the coordinates printed at the bottom of the screen.

E. If you wish to know the value of an area, move the marker to the lesser value of x, and press

$AREA_{menu}$

Then move the marker the the greater value of x and press

$AREA_{menu}$

again to observe the value of the area printed on the screen.

Note: If at any time ON takes you out of the plot menu completely, press orange PLOT then $PLOTR_{menu}$.

F. If you now wish to plot f'(x), press any key in the top row; then press

NEXT $f'(x)_{menu}$

and observe f' $(= 3x^2 - 1)$ plotted on top of f(x). (Any further instructions as to root, slope, etc., will now refer to f '(x)) When you are finished, press

ON $ERASE_{menu}$ NXT $RESET_{menu}$

This erases the graphs and resets all the plotting parameters to their default values.

IX. To sum a series, say $\sum_{k=1}^{100} \frac{1}{k^2}$, type:

' blue arrow button Σ K=1, 100, 1+K y^x 2 ENTER EVA

X. To obtain a Taylor series, say of sin x (up to and including x^5 terms), type:

orange arrow button ALGEBRA
' SIN X ENTER ' X ENTER 5 $\text{TAYLR}_{\text{menu}}$

XI. To plot a conic section, say of the circle $x^2 + y^2 = 1$, type:

'X y^x 2 + Y y^x 2 = 1 ENTER
orange arrow button PLOT $\text{STEQ}_{\text{menu}}$ $\text{PTYPE}_{\text{menu}}$ $\text{CONIC}_{\text{menu}}$
$\text{PLOTR}_{\text{menu}}$ $\text{ERASE}_{\text{menu}}$ $\text{DRAW}_{\text{menu}}$

XII. To plot a graph in polar coordinates, say $r = 1 + \cos\theta$ (from 0 to 2π), type:

'R = 1 + COS θ ENTER (θ is obtained by pressing α blue arrow F)

orange arrow button PLOT $\text{STEQ}_{\text{menu}}$ $\text{PTYPE}_{\text{menu}}$ $\text{POLAR}_{\text{menu}}$
$\text{PLOTR}_{\text{menu}}$ 'θ $\text{INDEP}_{\text{menu}}$ $\text{ERASE}_{\text{menu}}$ $\text{DRAW}_{\text{menu}}$

XIII. To plot a parametric graph, say $x(t) = \cos t$, $y(t) = \sin(t)$, from t = 0 to 6.3, type:

' COS T right triangle button + i * SIN T ENTER
(i is obtained by pressing α orange arrow I)

orange arrow button PLOT $\text{STEQ}_{\text{menu}}$ $\text{PTYPE}_{\text{menu}}$ $\text{PARA}_{\text{menu}}$
$\text{PLOTR}_{\text{menu}}$ orange arrow { } T SPC 0 SPC 6.3 ENTER INDEP
$\text{ERASE}_{\text{menu}}$ $\text{DRAW}_{\text{menu}}$ (note that SPC is the space button)

XIV. To create a subdirectory (let us call it "information"), type:

'INFORMATION ENTER
orange arrow button MEMORY $\text{CRDIR}_{\text{menu}}$

To see this, press
VAR $\text{INFOR}_{\text{menu}}$
To exit out of this subdirectory and go up, press
orange arrow UP

Note: When you are in a subdirectory, you can store anything by typing 'its name STO.

XV. To store an equation under a name (such as f = kx under HOOKE), type:

' F = K * X ENTER ' HOOKE STO

To see this, press
VAR $\text{HOOKE}_{\text{menu}}$

XVI. To find the vector dot (cross) product of two vectors (such as (1,2) and (3, −5)), type:
blue arrow MATRIX (if VEC_{menu} does not have a small box next to it, press VEC_{menu})

1 ENTER 2 ENTER ENTER blue arrow MATRIX
3 ENTER 5 ± ENTER ENTER MTH $\text{VECTR}_{\text{menu}}$ DOT_{menu}

(use $\text{CROSS}_{\text{menu}}$ if the cross product is desired)

LESSON 1: GRAPHING ON THE HP48SX

1. Turn the calculator $\boxed{ON}$ using the key in the (9,1) position where the indicates the 9-th row of keys and the 1 indicates the first column of keys. The box around ON indicates a key on the calculator.The zero (0 row is the menu row on the screen of the calculator and is indicated b underlining.

2. Let's graph y= f(x)= x^2-2x-1. I will use the * symbol for multiplication x to distinquish it from the variable x.

   ↰ 8 gives the PLOT menu.
   (7,1)

'	α	1/x	y^x	2	-	2	*	α	1/x	-	1	<u>STEQ</u>
(3,1)	(6,1)	(4,6)	(4,5)	(8,3)	(8,5)	(8,3)	(7,5)	(6,1)	(4,6)	(8,5)	(8,2)	(1,5

The function to be graphed is now stored. Now graph it.

<u>PLOTR</u>, <u>ERASE</u> <u>DRAW</u>
(0,1) (0,1) (0,2)

3. a. Change the x-Range from - 6.5 6.5 to - 2 4.
   $\boxed{ON}$ $\boxed{2}$ $\boxed{+/-}$ $\boxed{SPC}$ $\boxed{4}$ <u>XRNG</u>
   (9,1) (5,2) (9,4) (0,4)

   b. Change the y-Range from - 3.1 3.2 to - 4 4.
   $\boxed{4}$ $\boxed{+/-}$ $\boxed{SPC}$ $\boxed{4}$ <u>YRNG</u>
   (5,2) (9,4) (0,5)

   c. Graph f(x) with the new Range.
   <u>ERASE</u> <u>DRAW</u>
   (0,1) (0,2)

4. This calculator does not have a TRACE key but you can find an approximation for the y-intercept by using <u>COORD</u> (0,4). Move the + cursor on the x-axis with ▼ and ◀ to the y-intercept and press <u>COORD</u>. With the Range we are using we can only get the approximatio that the y-intercept is between -.9523 and -1.079 depending on where y have placed the cursor. The exact answer when x= 0 is y= -1 since f(0)=-1.

5. To get the menu back press any of the white keys.

6. Pressing <u>LABEL</u> (0,5) will label the x and y axis. Try it.

7. Pressing <u>CENT</u> (0,3) will regraph f(x) with the + cursor at the center of the screen. Move the + cursor to the vertex (bottom) of the parabol and press <u>CENT</u> . The Range values will change.

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8. Reset the Range to the calculators built in Range.
ON (9,1) NXT (2,6) RESET (0,6) then ↰ (7,1) 8 for the plot menu PLOTR (0,1) ERASE (0,1) DRAW (0,2)

9. Overlay f(x) with the graph of g(x)= x-2.
ON ↰ 8 for PLOT then ' α 1/x - 2 STEQ PLOTR DRAW

10. You can not find the intersection of the graphs with ISECT if they are just overlayed . You have to graph x^2 -2x-1=x-2 (see #2 if necessary) or {'x^2-2x-1''x-2'}. After the graphing is complete press FCN (0,6)

Move the + cursor using ◀ or ▶ to a spot near the intersection point in the first quadrant. Press ISECT (0,2) Ans.(2.618033...,.618033...)

Press ON This will put the coordinates on the stack. Move the cursor near the intersection point in the fourth quadrant. Press ISECT and ON. Press ON again and then ▲ which activates the interactive stack i.e. allows you to manipulate items on the stack. What are the actual intersection points? hint: Use the quadratic formula. Lesson 2 will teach you how to use the built in quadratic formula that the calculator contains.

You will notice when you go back to the stack with ON that some digits are off the screen. Do an EDIT with ↰ +/- (5,2). You have to have the stack cursor ◆ at the level you want to investigate. Use Skip → (0,2)▼ to get to the end of the y-coordinate. Press ON move the stack cursor ▶ to level 2 then press RollD (0,4) to put the other coordinates at level 1, move ▶ back to level 1 and then use ↰ Edit +/- (5,2) Skip → (1,2)▼again to see the other y coordinate.

LESSON 2 ON THE HP48SX:QUADRATIC FORMULA

Let us solve the equation of lesson 1 with Quad. We had $x^2-2x-1=x-$ which is equivalent to $x^2-3x+1=0$.

1. On the calculator type 'x^2-3x+1 ENTER `x ENTER ← 9 for algebr and press QUAD at (0,4). You will see 'x=(3+s1*2.2360679775)/2'. The s stands for +/-1 in front of the radical in the quadratic formula. .Not this is a small case s. To get a small case s on the calculator you hav to press ← after you press α. To evaluate for both +1 and -1 do th following . First make a copy of what is on the stack by pressing ENTER. Now type 1'α← s 1'STO(3,2) (you'll have to use > to get past the ' befor you press STO);then press EVAl (3,3). After writing down the answe 2.61803398875 press ⟵ at (5,5) to get back to the expression . No enter 1 +/- 'α← s1 ' STO EVAL for the second root of the equatior .38199996601125.

There is an alternative to using Quad and it can be used fo evaluating any equation with any number of variables and it has a simple method to deal with the substitutions for s1.

Repeat the first two lines of #1 up to the expression for x=. **Now dc ← 7 for SOLVE and then press STEQ (0,5) followed by SOLVR (0,1). Or the bottom of the screen you should see x,s1,EXPR=. PRESS 1 and then the S1 (1,2) and then EXPR= (1,3)to get x=2.618033988.

Then enter a -1 FOR S1 AND press EXPR= to get x=.38196601125. Compare these values with the ones we got above.

If perchance you do not get the correct values check the following. clear the stack with → ⟵ for Clr (5,5) which clears the stack. Then do → HOME (4,1) VAR (2,4). Your HOME should be empty at this point . If it isn't then use braces ( ) and tap the white keys to put the items in HOME between the brace brackets. Then do a ← PURGE (5,4) to get rid of the superfluous items. Use ←7 for Solve press and return to ** above.

LESSON 3: PROGRAMMING ON THE HP48S

The program markers are << >> (8,5) i.e. ↰ −.

This program will evaluate a given f(x) for any value of x that you choose. If you have not created a directory for Calc 1 please do that first, 'CALC1' ↰ MEMORY (2,3) CRDIR (0,5). Before you start writing the program make sure the top of the screen says HOME CALC1 so when you store the program it will be in the CALC1 folder.
This is the program stored with the name VALF.

```
<< 'X' STO DUP EVAL SWAP 'X' PURGE >>

    ENTER 'VALF.' STO
```

You can spell out the words or get them from the calculator.

To get DUP: PRG (2,2) STK (0,1) NXT (2,5) DUP (0,1). The others are on the keyboard i.e. STO (3,2),EVAL (2,3),SWAP (3,6), PURGE (5,4)

To run the program enter f(x) on the stack then enter a value for x on the stack and then press VALF on the menu board. Always do a simple problem first to see if your program is working correctly. Enter '2x+1' on the stack and then enter 3 on the stack and hit VALF. on the menu bar and you should see the answer 7 above the function. You can keep doing this for as many values as you need or want.

If you want to see the program or change it press 'VALF.' Enter visit (5,2).

LESSON 4: GRAPHING PIECEWISE DEFINED FUNCTIONS ON THE HP48SX

LET'S GRAPH
$$F(X)=\begin{cases} .6X+4.5 & \text{IF } X<-2.5 \\ 2+\text{SIN } X & \text{IF } -2.5\leq X<2.5 \\ -\text{COS } 2X & \text{IF } X \geq 2.5 \end{cases}$$

'IFTE (X<-2.5, .6*X +4.5, IFTE (X<2.5,2+SIN (X),-COS(2*X))'

IFTE IS AN ANACRONYM FOR IF THEN ELSE. I use the word otherwise.

IFTE IS FOUND BY PRESSING PRG (2,2) BRCH (0,5) NXT NXT (2,6) IFTE (0,4

< is found by pressing , PRG (2,2) TEST (0,6) NXT(0,3). NOW GRAPH AS YO WOULD ANY OTHER FUNCTION MAKING SURE YOUR CALCULATOR IS NOT IN CONNECTED MODE. TO BE IN DISCONNECTED MODE ↰ MODES (2,3) NXT CHECK THAT CNCT DOES NOT HAVE THE LITTLE SQUARE BY IT. IF IT DOES ,PRESS THE D WHITE AND THE BOS SHOULD APPEAR. NOW GRAPH STARTING WITH ↰8 FOR PLOT then STEQ PLOTR,ETC. .

ANOTHER EXAMPLE:

GRAPH
$$h(X)=\begin{cases} -2X & \text{IF } X < 0 \\ X^2 & \text{IF } 0 \leq X < 1 \\ 2 & \text{IF } X \geq 1 \end{cases}$$

'IFTE(X<0, -2*X, IFTE (X<1, X^2,2)'

HP-48 Lesson 5

PROGRAM TLIN

The program TLIN will give the equation of the tangent line to f(x) at any point you choose.

Tangent Lines (TLIN)

Put this in your Calc I folder

```
<< 'X' STO DUP 'X' ∂ 'X' X - *
SWAP EVAL 'Y' SWAP - SWAP = 'X' PURGE >>
```

EXP: Find the equation of the tangent line to the curve $y = x^2-3$ at point (2,1)

To run the program:

Put f(x) on the stack

Put the x-value 2 on the stack

Press T-LIN

Ans: 'Y-1 = 4*(X-2)'

Paper way: Find the derivative of y for $y = x^2-3$

$$y' = 2x$$

so slope at 2 is 2(2)= 4

$$y-y_1 = m(x-x_1)$$

$$y-1 = 4(x-2)$$

To graph f'(x) graph f(x), FCN, NXT, F' will graph the function and its derivative. The derivative gets plotted first.

Neat: If you put the cursor on a curve and press NXEQ the equation will appear on the screen. If you put the cursor on a curve and press f(x) you will get the y-value for the function at that point.

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PROGRAM IMPD. FOR IMPLICIT DIFFERENTIATION & INTRODUCTION TO THE EQUATION WRITER

PUT THIS PROGRAM IN YOUR CALC 1 FOLDER.

```
<<DUP 'Y' ∂ COLCT 'FY' STO DUP 'X' ∂ NEG COLCT FY / COLCT 'FY' PURGE>>
```

COLCT IS ← ALGEBRA (6,4) (0,1)

TO RUN THE PROGRAM PUT THE EXPRESSION ON THE STACK. PRESS IMPD.
EXP: X^2+Y^3-5 ENTER IMPD
THE ANSWER ON THE CALCULATOR '-'(.67*X*Y^-2)

ON PAPER: 2X +3y^2=0 ⇒ 3Y^2 DY/DX=-2X ⇒ DY/DX =-2X/3Y^2 WHICH CAN BE WRITTEN AS (-2/3) X y^{-2}=.667XY^{-2}.

HP48SX - Lesson 7
Program to do Newton's Method

Create a directory for Newton in your Calculus I Folder

'NWTN' ↵ Memory CRDIR

You'll need to put the following program in first. It is to be use *before* running the program NEWT. Its purpose is to store the current function and its derivative for use by the Newton-Raphson algorithm.

PROGRAM FSTO:

```
< < DUP 'F' STO 'X' ∂ 'FPR' STO > >
  ENTER  'FSTO' STO
```

Put the following in the NWTN folder also. This program requires a initial value of X (first guess) to be on the stack before execution.

$$Xn+1 = Xn - \frac{f\ (x)}{f'(x)}$$

PROGRAM:NEWT

```
   < < 'X' STO X X F EVAL FPR EVAL
÷ - 'X' PURGE > >

ENTER 'NEWT' STO
```

1. Note: To get an initial value you could graph f first.

2. Note: You have to keep pressing NEWT after you get your initial value to get better approximations.

Using an HP-48G -- Some Calculus Examples

The HP48G & GX are the improved, more user-friendly versions of the HP48S & SX. In each series, the X model is expandable, that is, it has two slots which can be filled with program cards or memory cards. Three models have 32K memory; the GX has 128K built-in memory. Many of the S series features still appear on the G series. This lesson will acquaint you with some of the common and additional features.

For convenience, boxed words represent actual keys; underlined words represent "soft" keys -- these are menu options which appear at the bottom of the screen over the white keys. Pressing a white key selects the menu item directly above it. Menu items which have a tab on their top left like a file folder are exactly that -- a folder which contains an additional menu.

Also note, on the calculator, each key has at least four uses. Its immediate usage is on its face. A white letter of the alphabet appearing to the bottom right must be preceeded with the alpha key. Above each key are two words, one green, one purple. Words in purple require that the purple arrow (shift) key be pressed first; words in green require the green arrow first. Combinations of alpha with either green or purple will produce the Greek alphabet, lower case English alphabet, and special symbols.

As this lesson proceeds, I assume you will remember some things, so I will leave out some of the detail.

1) Computing a limit

HP calculators all work using reverse Polish notation, that is, arguments or operands first, operations last.

First, let's type in an expression

$\boxed{'}$ $\boxed{\alpha}$ $\boxed{X}$ $\boxed{y^x}$ $\boxed{2}$ $\boxed{-}$ $\boxed{9}$ $\boxed{\text{ENTER}}$ $\boxed{'}$ $\boxed{\alpha}$ $\boxed{X}$ $\boxed{-}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ $\boxed{\div}$

You noticed that 'X^2 - 9' started on the command line, then moved to line one when you pressed $\boxed{\text{ENTER}}$. 'X - 3' appeared on line one after the second $\boxed{\text{ENTER}}$ while 'X^2 - 9' moved up to line two. After the $\boxed{\div}$ a fraction was formed on line one with parentheses added.

a) using the Solver

type $\boxed{\text{green arrow}}$ $\boxed{7}$ choose "SOLVE EQUATION" by pressing <u>OK</u>

Now to get the expression we just typed in press $\boxed{\text{NXT}}$ <u>CALC</u> <u>OK</u>
If something other than our fraction appears on line one after you pressed <u>CALC</u>, press $\boxed{\text{purple arrow}}$ $\boxed{\text{DROP}}$ before the <u>OK</u>

we should now see '(x^2 - 9)/(x -3)' in EQ:

use the down arrow to highlight X and type in 2 OK
press EXPR= you won't see anything happen.
Highlight X again by presing the down arrow ▼ .
Now type in 2.1 for X and press OK ; press EXPR= again
Press ON and you'll see on the screen EXPR:5
EXPR:5.1

b) Press VAR
notice EQ on the VAR menu -- press that and our expression is back. (You may have to press NXT once or twice until EQ is visible on the menu line.)

A better way to get a table of values is to define a function

First type in 'F(X)' by pressing ' α F purple ( ) α X ENTER SWAP =

Pressing purple-arrow DEF will create a function key on your VAR menu; this works like any other function key e.g. SIN, COS, etc. Again, you may have to press NXT a few times to find the F on the menu line. Note this is not the same a the white key labeled F -- unless by coincidence, your defined function F lands above the F white key !

Let's compute $\lim_{x \to 3} \frac{X^2 - 9}{X - 3}$

by computing the values for 2.9, 2.99, 2.995, 3.1, 3.01, 3.0015
Type in 2.9 ENTER then press the white key under F to see 5.9 appear on the screen. Repeat this for the other numbers and you'll see a table of function values extending up the stack.

2) Lets graph that function as well press green-arrow 8
Check that the TYPE: is FUNCTION. If FUNCTION doesn't appear on your screen, highlight TYPE using the up ▲ or down ▼ arrow.
Press CHOOS from the menu.
Highlight FUNCTION and press OK.

Notice the EQ: is the same as before; check to see the screen settings are -6.5 to 6.5 horizontally, -3.1 to 3.2 vertically and that the INDEPendent variable is X. To change any of these, just highlight it, type in what you want onto the command line, and press OK. To get a negative number, remember you must type the number first, the +/- second.

Note that the numbers I gave above are the standard settings (default) and always give what is called a "friendly window" to match the screen's 131 pixels horizontally and 64 pixels vertically.

When everything is set, press ERASE DRAW

Not high enough? press ON and change V-view to a maximum of 7 by highlighting the rightmost V-view number, typing in 7 and pressing OK
ERASE DRAW again. We can even see the missing pixel over x=3

Another possibility is to choose the ZOOM menu, then choose VZOUT (after pressing NXT once.)

Once we have the graph on the screen, we can activate TRACE first, then (X,Y)

When you're done, press ON a couple of times to return to the main screen.

3) Symbolic Algebra

Press green-arrow 9 . Use the ▼ to highlight through the choices and choose MANIP EXPRESSION by pressing OK

We need an EXPR so choose EDIT and type in '(x-2)*(x+3)*(x-1)'
EDIT supplies the quotation marks; if you don't press EDIT first you just have to type in your own '

' purple () α X - 2 then use the large right arrow ▶ to move the cursor over the parenthesis before continuing

* purple () α X + 3 ▶ * purple () α X - 1 OK

Press EXPN three or four times, then COLCT then alternate them a few times;
we should see '-6 + X^3 -7*X'

We'd like to graph that so press NXT CALC ENTER OK ON

You'll notice our expression is on the stack. The ENTER duplicated it; one copy remained on the stack; one was returned to the EXPR

4) green-arrow 8 again. To retrieve that expression we just worked on, type NXT CALC DROP OK

This got rid of the old expression (that fraction from part 2) and retrieved the new one from the stack

Now press NXT ERASE DRAW and we should see a lovely graph

Let's investigate FCN . You'll notice it has a tab on the top left; there are more goodies hidden in there for us to play with.

Using the four arrow keys, move the crosshair to the right on the curve; press ROOT . Note the answer appears on the screen as ROOT:1. Press any white key to return the menu to the screen. Now find the SLOPE at the point; this time SLOPE:-4 appears at the bottom of the screen; again press any white key to retrieve the menu. Press NXT once to find TANL . Note you will see the equation of the tangent line on the screen after you press TANL, in addition to graphing it

EXTR will find the extreme point near the current cursor position.

Let's return to the main screen for a minute by pressing ON ON. You'll see all the information you just found on the screen with labels.

If you'd like to tour the stack, press ▲ and notice that a large arrow cursor has appeared on line one. ▲ will continue up the stack as far as you want. You'll notice that the menu line has changed as well. You can ROLL pointed to items up or down the stack, PICK them off and place on line one. →LIST will even make a set including everything from the pointer down to line one.

To de-activate the stack pointer, press ON

Let's redraw out picture, so press green 8 then ERASE DRAW Press ZOOM and the ZIN this time. Now press FCN. Move the crosshairs to the left this time and press EXTR -- note that we can find a max or min even it it's off the screen

F'(X) will redraw the graph with its derivative superimposed use TRACE and (X,Y) to move along both curves. ◀ ▶ will move along the current curve; ▼ ▲ will switch to the other curve.

Also under FCN, VIEW will tell you which curve you are on NXEQ cycles the order of the curves. F(X) gives the value of the function at the current X-position of the crosshair. Using a combination of NXEQ and F(X) is a nice way to show corresponding points on a function and its graphed derivative.

Move the crosshair to a convenient spot, press SHADE to anchor an x, then move the crosshair to the right; pressing SHADE again will shade between the two curves.

When we press ON we return to the plotter screen. Notice that the EQ has changed. If you highlight EQ and press EDIT you'll see something new -- a set containing the original function and its symbolic derivative. Remember that the HP has symbolic capabilities, so when we press F'(X) a while ago, we not only got the graph, but we also generated the actual function derivative. What NXEQ was really doing when we used it, was to cycle the two functions within the set.

Press ON again to return to the main screen. Notice that the many things we found ar: on the stack with labels. If you'd like to clean off the stack, use $\boxed{\text{purple}}$ $\boxed{\text{CLEAR}}$ However, the stack feature is a handy way to retain all that you've worked on without writing it down at each step. The stack can be filled up to hundreds of levels -- until all the HP's memory is used up.

5) POLAR graphs
first, lets go to $\boxed{\text{MODES}}$ and choose a polar coordinate system. Which steps must you follow? also be sure to choose radians.

$\boxed{\text{green}}$ $\boxed{8}$ again, choose POLAR for the type

Now let's type in { 'sin(2*T)' 'sin(T)'} for EQ: These are two items in set brackets. Notice that once you have pressed the $\boxed{\text{'}}$ and then press $\boxed{\text{SIN}}$ the parenthesis are supplied for you. Remember to use the right arrow ▶ to get out of the)' of the first function before you can start the second function. Then change the independent variable to T

Select OPTS to check the values for T, which should be automatically set to 0 to 2π . Also let's graph simultaneously; when you highlight SIMUL you will see ✓CHK on the menu below. Press it, then $\boxed{\text{OK}}$

Also change HView to -1.5 to 1.5, VView to -1 to 1
ERASE DRAW

Notice that TRACE (X,Y) gives the T value and goes around either curve

A Word of Caution: If you set the independent variable to HI and LO values under OPTS, be sure to RESET them later if you do other graphs. Those values can come back to haunt you!! It's possible to have the H-View set differently from the HI and LO for the independent variable, and so get only part of a graph. Frequent causes for strange graphs you didn't expect are 1) degrees when you really want radians; and 2) left-over settings on HI and LO on the OPTS screen.

6) Parametrics: change the plot type to PARAMETRIC

Suppose the person of your dreams is on a ferris wheel that has a radius of 20 feet and makes one revolution every 12 seconds. You're 75 feet away from the center of the ferris wheel, and you throw him/her a token of your affection when he/she is at the three-o-clock position on the wheel. Since you're a mathematician and a whiz at calculus, you know the initial velocity of the package is 60 feet/sec and your arm is at an angle of elevation of 60 degrees.
How close does this thing come to your friend?

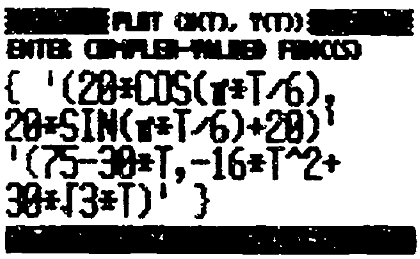
Parametrics are entered as complex numbers of the form (x(T), y(T)) so the equations are

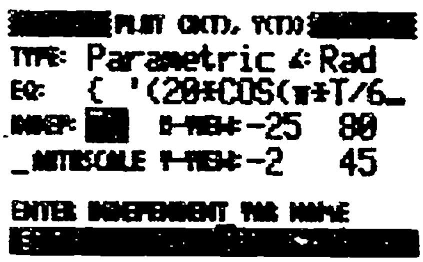
wheel: '(20*cos(*T/6), 20*sin (*T/6) +20)'

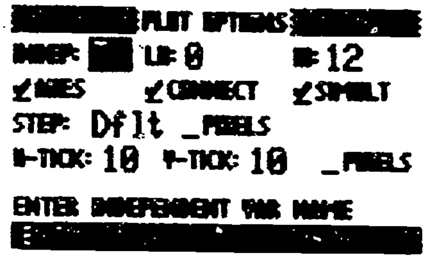
object: '(75-30*T, -16*T^2 + 30* 3 *T)

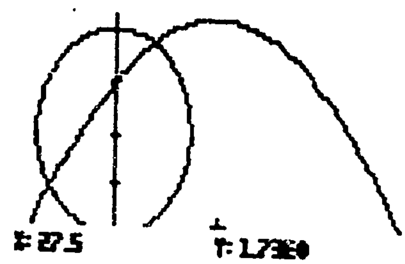
we can enter them into EQ: using set notation { ...}

the plot screen and options screen should be set as shown in the following pictures.







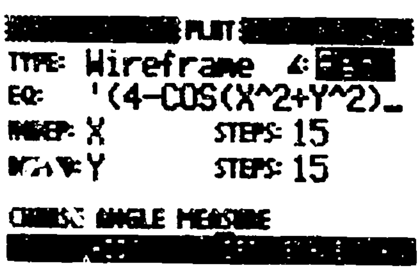


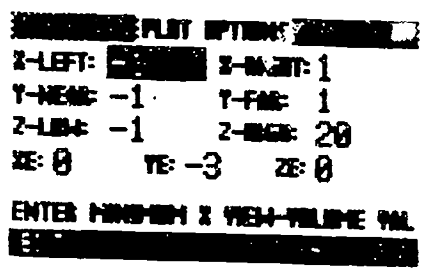
7) three dimensional plots!!!

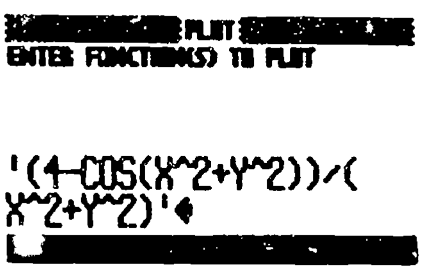
green 8 and change the plot type to WIREFRAME

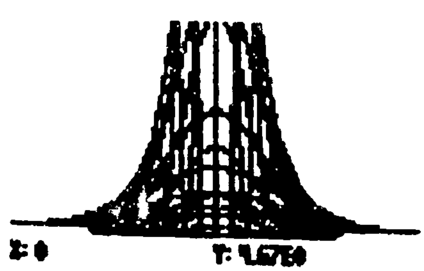
for a function of two variables try '(4 - cos(X^2+Y^2))/(X^2+Y^2)'

other settings are shown below









BEST COPY AVAILABLE

8) EQUATION WRITER lets you type in things exactly as you would write them on the board. When you press purple EQUATION you'll see a blank screen with a small oval cursor on the middle of the left side.

Type in green ∫ . Notice the oval is now at the bottom of the integral sign; it won't move until you type in the lower limit. How about 0? The oval is waiting in case you wanted a two or three or more digit number. Press ▶ and the oval moves to the top of the integral sign for the upper limit.

Type 1 ▶ then √ α X ▶ purple ( ) 1 + 1 ÷ 4 α X ▶ (this gets you out of the denominator) ▶ (out of the square root) ▶ (out of the expression) ▶ (provides a d) now type α X ENTER.

$$\int_0^1 \sqrt{X} \cdot \sqrt{\left(1+\frac{1}{4 \cdot X}\right)} \, dX$$

[redacted]

As you type in each object in the expression, the right arrow ▶ key moves you along through the expression. Should you make a mistake along the way the right-pointing arrow key which is also labeled DROP will back up over the thing you typed; howver it is painfully slow -- all this is actually graphics, so it all has to be refigured internally and redrawn.

You'll notice the item on line one of the stack looks different from what you last saw, but the notation is logical. You could have just typed it in like that. Now press →NUM to get the value of the definite integral.

You can also do indefinite integrals as $\int_0^T \mathrm{SIN}(X)\, dX$

[redacted]

Use EVAL . These can also be done in the SYMBOLIC menu green 9

9) Back to the Solver; press green-arrow 7 then highlight SOLVE POLY...
press ENTER

Type the coefficients [1 5 6] OK This must be in matrix form, and represents the polynomial X^2 + 5 X - 6
Now press SOLVE to get [-2 -3] press SYMB
You'll see nothing new now but
when you press ON or NXT OK , on the stack you'll see

2. ROOTS: [-2 -3]
1. '(x+2)*(x+3)'

10. How about green-arrow 9 choose INTEGRATE

Type in 'SIN(X)' 'X' for the variable LO:0 and HI: T you can choose a SYMBOLOC (or for another example NUMERIC) result

SYMBOLIC gives you an answer that looks like the Fundamental Theorem of Calculus. EVAL then gives the answer '- COS(T) + 1'

11. this time in green-arrow 9 let's highlight TAYLOR

SIN(X) is still stored as the EXPR: from your integration above.

We'll do this twice to get the Taylor polynomials of degrees 5 and 9. We'll need the VAR to be X, order 5. When SYMBOLIC is highlighted just press OK.
Now repeat the process for order 9.

Both will be on the stack. Use the up arrow ▲ to activate the stack, point to the polynomial on level 2 and then press →LIST.

This puts both items into a set at level one. Press ON to deactivate the stack pointer. Now let's type in 'SIN(X)' ENTER + to add the sine into the set.

To plot, press green-arrow 8 be sure we have FUNCTION now highlight EQ
press NXT CALC DROP OK to put the list into EQ:

Be sure the screen in correct; be sure RAD is set. Also check OPTS now ERASE DRAW

Having the calculator set to SIMULTANEOUS mode is very effective since you can see the curves join and diverge again as they are graphed. If sequential mode is set, you may wish to put the sine first in the list to see where the others join it.

12. Take a peek at EQ LIB to see the goodies there.